Liquid Penetration Modeling for Cloth Dyeing

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Abstract

This paper presents a model of cloth dyeing using the characteristics of the thread and weave pattern. The proposed dyeing model is based on Fick’s second law that defines the molecular transfer under translational diffusion [Fic85]. The algorithm in the proposed model calculates the dyeing distribution from parameters such as the amount of dyeing, saturated amount, and pressure in each cell on a timeline. We improve the algorithm based on Fick’s second law to consider a woven cloth structure and describe the proposed model of the structure of woven cloth as a two-layer cellular model. We then visualize the cloth using a simple 2D shading method of asperity by using the color distribution on a dyed image of real woven cloth. In addition, we provide a method for producing dyeing patterns without dyeing diffusion. The proposed method produces images that capture several of the characteristics of dyeing observed in real dyed cloth.

1. Introduction

Dyeing is a traditional method of imbuing a cloth with color. It is a traditional art form that produces interesting patterns, not only according to the artist’s intent, but also unaccountable physical factors. However, liquid penetration into cloth is a complicated phenomenon from a scientific and physical standpoint, and the pattern and color of dyed cloth is a function of the physical properties of the dye and the fabric. In other words, dyeing cannot be completely controlled by the fabric-dyeing artist. Predicting the actual result of dyeing using graphics would be useful for designing cloth with accuracy, especially since the dyeing process is a troublesome task in reality. More people would be interested in the dyeing process if they could better predict the final product.

Most existing non-photorealistic rendering research has examined fine arts such as painting and drawing. In this area, images are generated with paper and canvas textures. Dyeing is an art equivalent to painting, with cloth as the medium instead of canvas or paper. In order to characterize dyeing, physical models of cloth dyeing and models representing the texture of cloth are essential. Furthermore, we must construct a design system to represent the art form of dyeing, such as creating patterns on cloth with dye. Therefore, we propose a comprehensive model of these elements, which are necessary for the visual simulation of dyeing. The present paper is organized as follows: The following section presents related work. Sections 3 and 4 describe dyeing and the visual characteristics associated with the technique. Section 5 describes dyeing simulation, and Section 6 presents the results of the present study. Section 7 then describes program usage, and Section 8 presents the conclusions of the present study and describes areas for future study.

2. Related Work

The methods for simulating painting tools and drawing strokes are progressing in the area of non-photorealistic rendering (NPR). Some research in this area includes watercolor painting and Chinese ink painting, both of which involve the diffusion and pigment density color using paper [GK91] [CAS*97] [ZSyT*99] [Lee01] [GK03] [CT05]. Curtis et al. [CAS*97] presented a method for watercolor effects such as dry-brushing, intentional backruns, and flow patterns. Their model simulates three phenomena: the flow of water on paper, the flow through paper fibers, and pigment deposition. They successfully simulated watercolor paint on paper. Chu et al. [CT05] developed a novel method for rep-
representing the real drawing of a fluid on absorbent paper with the Lattice Boltzmann equation in real time.

There has been some research on methods, such as the batik method, that can be employed for painting on cloth. Wyvill et al. [WvOC04] presented an algorithm for simulating the cracks found in batik. Their method can produce convincing patterns that capture many of the characteristics of the crack patterns found in real batik cloth. Also, Drago et al. [DC04] performed simulations of the canvas used for easel paintings. Their work represents real woven canvases and considers weaving patterns and canvas aging. However, the texture of the cloth is not included in Wyvill’s algorithm, and the diffusion of paint in cloth is not considered in either Drago’s or Wyvill’s research.

There has also been some research on the methods for the visualization of cloth. These studies attempted to represent interwoven threads and to model the interaction of light with visualization of cloth. These studies attempted to represent the threads in detail [WAT92][XCL*01][SYO04]. However, this level of detail is not always necessary and we therefore focused on the structure of the cloth for diffusion. We therefore propose a very simple rendering method for cloth that is suitable for representing dyeing.

To simulate dyeing, it is necessary to consider the weave structure of the warp and weft. In this research, we represent dyeing by the interaction of weave patterns and the diffusion of dye.

3. Outline of Dyeing

The purpose of dyeing is to change the color of cloth, that is, to bond dye tightly to fabrics. The key elements of dyeing are cloth, dye, and dyeing techniques, each of which consists of numerous finer aspects.

Cloth is a fabric structure woven from threads, which are in turn composed of fibers. Since the dye diffuses along these fibers in a cloth, the directional characteristics of the thread have a marked affect on dye diffusion into the cloth. The two-directional thread characteristics of woven cloth are referred to as the weft and warp and the type of cloth is classified according to its weave pattern. The most well known weaving pattern is a plain-woven pattern. A plain weave is the style of weave in which the warp alternates over and under the weft. However, in addition to the weave pattern, there are other elements that affect dyeing, such as the interval between fabrics. Narrow intervals between fabrics prevent dyeing [Okik94][Kur96][Sak99][Yos02].

Since dyeing techniques are important for determining how dyeing is applied as an art form, we describe the following common dyeing process using the example shown in Figure 1 and described below. 1) Prepare the cloth by cleaning. 2) Prevent parts of the cloth from dyeing to create a pattern. The two techniques of tie-dyeing, which adds pressure to the cloth with folding and sewing, and batik, which covers the cloth with wax, before the dying process. 3) Wet the cloth with water. 4) Dip it in the dye compound. 5) Dry and open it. 6) Wash the cloth to remove dye pigments which are not firmly fixed into the fabrics, and 7) dry it. After drying, the dyeing process is finished.

Figure 1: Outline of the actual dyeing process. The image on the left shows the cloth before dying (1), the center image shows the cloth after being tied (2), and the image on the right shows the cloth after the entire dyeing process (3,4,5,6,7).

4. Visual Characteristics of Dyeing

In this section, we describe the visual characteristics of dyeing caused by the following three physical elements.

1: The "dye stain shape" and "mottles" caused by the directional characteristics of the dyeing of thread. The "dye stain shape" changes as the amount of dye changes. It approaches a circular form as the amount of dye increases and a rhombic form as the amount of dye decreases. Figure 2 (a) shows the different shapes of dye stains depending on the amount of dye. In this figure, the amount of dye decreases from right to left. "Mottle" refers to the extreme deep sections of color around the edge of a stain and is strongly affected by the directional characteristics of the thread. Cloth consists of interwoven and piled weft and warp, which enables cloth to be described as a two-layered material. It is important to produce mottles in dyeing because the layers in cloth reduce the diffusive conditions between them. Thus, the two-layered cloth model can be used to represent mottles. Figures 2 (b) and 2 (c) show a magnified image of a stain, and Figure 2 (c) shows a magnified image of a stain with its gray values exaggerated.

2: An arrangement of square elements appears due to the shades of the threads: That is, the shades of threads within the cloth look like an arrangement of square elements. We can also see that characteristics in Figure 2 (b). These square elements can fluff up the fabric and generate a fuzzy appearance.

3: Gradation is caused by pressure dispersion: There are some dyeing techniques in which an application of pressure on cloth prevents it from dyeing and therefore makes patterns on the cloth. The pressure adds a part of the cloth disperses and smoothes the values of stain. Figure 2 (d) is a magnified image of dyeing with pressure. The pressure smoothly disperses and creates a gradation of the color.
5. Dyeing Simulation

In this section we describe the proposed methods for modeling cloth and dyeing diffusion and for the visualization of the above characteristic phenomena.

5.1. Cloth Model

In a previous study on NPR, paper and cloth were represented using a two-dimensional model. There is an advantage to reducing the computational cost in the two-dimensional paper model. In addition, since the model is capable of simulating painting it is thus suitable as an artistic tool. However, as described in Section 4, a two-dimensional model is not well suited to representing dyeing characteristics. In the proposed model, we introduce a two-layered cloth model in order to minimize z-axis factors.

We use a cellular model to represent the cloth. First, we make a thread model by arranging the cells. Second, we construct cloth with thread models corresponding to the weaving pattern (Figure 3). The cells are then modeled as cloth, such that their size is equal to the width of each thread which has a different diameter. We use a random distribution for the diameter of wefts and warps for a more natural representation.

First, diameter \( w_1 \) (for the weft) and \( w_2 \) (for the warp) are defined at the user’s discretion. Next, the proposed model decides all values of the diameter of each weft and warp in a random range that is based on \( w_1 \) and \( w_2 \). The weft and warp are then constructed into a two-layer structure, as shown in Figure 3. We set these two layers in the cloth model and both of them appear according to the weaving pattern. Also, we simulate a woven cloth based on this cellular cloth model for dyeing and shading.

![Figure 2: Characteristics of cloth dyeing.](image)

5.2. Dyeing Model

We assume that the dye diffuses into the plain-woven cloth. Figure 3, cells that are dyed from pixel to pixel are defined as diffusion cells. We assume that the cloth is wet in the model according to an actual dyeing process, so its physical environment involves dipping the cloth into the dye.

In the dyeing system, the proposed cellular model decides whether each pixel belongs to the weft or the warp. Then, diffusion dyeing occurs on each pixel depending on its properties. We extend the diffusion equation to include weaving patterns; this original equation is Fick’s second law, which represents the molecular transfer under the condition of translational diffusion. We show the process of dye diffusion below.

1. We set the values of the parameters on all pixels.

   <Parameters on a pixel>

   - Concentration of physical quantity of dye: \((c\text{ in weft, } s\text{ in warp})\)
   - Position in the weft or warp: \((i, j)\)
   - The number of time steps: \((t)\)
   - Interval time for one step: \((\Delta t)\)
   - Interval between pixels: \((\Delta d\text{ for x and y-axis, } \Delta z\text{ for z-axis})\)
   - Diffusion coefficient: \((D_c\text{ in weft, } D_s\text{ in warp})\)
   - Pressure: \((P_c\text{ in weft, } P_s\text{ in warp})\)
   - The minimum saturation a pixel must have before it can diffuse to its neighbors when the cloth is dry to begin with: \((\varepsilon)\)
Pressure affects the diffusion coefficient and capacity as we describe below in this section. If ε is 0, we suppose that the cloth is wet in advance. There is another parameter that is adsorption rate a to stop the diffusion, which is a constant rate in all cells. These parameters represent the difference between wet to dry cloth.

2. The user indicates the dyeing areas by drawing or inputting an image. We define the dyeing areas in a dyeing table. Also, the user can indicate areas where pressure is applied in a way that is similar to having pressure being applied to the cloth. This pressure was defined using pressure tables, which indicate where each function is being applied. In addition, these tables indicate the degree of pressure and the amount of dye. However, in the case of generating a stain, the user indicates a point on a screen. Then, the cells in a certain range are saturated according to the amount of dye. This process does not occur without adding pressure to the design.

3. If the process includes pressure on the cloth as a dyeing technique, the pressure value is decided by the pressure distribution as shown in Figure 4. First, RGB values are smoothed by linear interpolation. Second, the values exceeding a certain value are set as maximum pressure 0.0, where minimum pressure is 1.0. The max is 0.0, which indicates that no dye is bonded. The minimum RGB values are set as the minimum pressure, 1.0. For simplicity, the RGB values between the maximum and minimum are converted to pressure values in the 0.0 to 1.0 range to adjust the diffusion coefficient such that a natural pressure dispersion is created.

4. Dye diffusion occurs in the threads. Dye compound is applied where it is indicated in the dyeing table. The amount of dye is user defined and is also defined by converting the RGB values from the dyeing table. Next, dye diffusion begins, with diffusion based on the formula of Fick’s second law:

$$\frac{\partial \phi}{\partial t} = D \frac{\partial^2 \phi}{\partial d^2}$$  \hspace{1cm} (1)

where $D$ is diffusion coefficient, $\phi$ is a concentration of diffuse matter such as dye. And discretized form of Formula (1) is

$$\phi_{i,j}^{t+1} = \phi_{i,j}^t + \Delta t \left( \frac{\phi_{i+1,j}^t - \phi_{i,j}^t + (\phi_{i+1,j}^t - \phi_{i,j}^t)}{2\Delta d^2} \right)$$  \hspace{1cm} (2)

Defines the one-dimensional molecular motion under the condition of translational diffusion. Dye diffusion is based on one-dimensional diffusion on the thread to be affected by the dimensional characteristics of the thread. In addition, the diffusion can be assumed to affect crossing threads. Therefore, we improve formula 2 so as to consider the diffusion to the crossing threads:

$$\phi_{i,j}^{t+1} = \phi_{i,j}^t + \Delta \left( \frac{D_1 (\phi_{i+1,j}^t - \phi_{i,j}^t) + (\phi_{i+1,j}^t - \phi_{i,j}^t)}{2\Delta d^2} + D_2 (\phi_{i+1,j}^t - \phi_{i,j}^t) \right)$$  \hspace{1cm} (3)

where $D_1$ is the diffusion coefficient in the same thread and direction as the thread of interest, $D_2$ is the diffusion coefficient in the cross thread in the z direction. We can represent various weaving patterns using the above formula, as illustrated in Figure 5. Furthermore, we define the diffusion coefficient $Dc$, $Ds$ randomly. This is different from the diffusion coefficient $D$ in the original diffusion equation, which is a constant. However, $Dc$, $Ds$ varies with position and the properties of the cloth. Since the diffusion coefficient varies with location in a real cloth, each pixel is assigned a different value. Also, the diffusion coefficient is affected by the degree of pressure because pressure prevents diffusion. Therefore, we can compute $Dc$, $Ds$ by setting:

$$D_1 = \left( \frac{Dc}{2} \right) \left( \frac{Pc_{i+1,j} + Pc_{i,j} + Pc_{i-1,j} + Pc_{i,j}}{4} \right)$$  \hspace{1cm} (4)

$$D_2 = \left( \frac{Ds}{2} \right) \left( \frac{Pc_{i+1,j} + Pc_{i,j}}{2} \right)$$  \hspace{1cm} (5)

$Dc$ and $Ds$ are set according to the diameter of each thread. The capacity of dyeing of a cell depends on the pressure value. Therefore, dye diffusion does not occur at all if the pressure value is 0.0 in the proposed system. Also, we can change $Dc$ and $Ds$ by multiplication with arbitrary decimal figure when position of the weft of warp in z-axis is changing.

In the proposed dyeing model, we do not set the amount of water and dye; instead, we set the concentration of the physical quantity of dye $c$ and $s$. This means that water saturates the space where the dye is nonexistent. In the case of dyeing dry cloth, we estimate dyeing using the same model, but assume that the water is air and add a rule for the surface tension of the liquid that stops the diffusion where the cell has insufficient dye. We represent the absorption of dye on fabrics by fixing absorption on a timeline at a constant rate. Also, the proposed model represents dyeing unevenness. A different diffusion coefficient for the bias of the fabric density and molecular concentration is used for dyeing unevenness. We use random numbers to determine the diffusion coefficients for each thread (Figure 2 (c)).

The following is an example of the properties we employ when we use a drop of dye on dry cloth with this model. We set double capacity of dye in cells inside the area of a circle with a radius of 30 pixels. We then apply the proposed dyeing model with properties such as those shown in Table 1 (1) to produce the result shown in Figure 10 (a). Conversely, the stain produced by dye diffusion on wet cloth produced using the properties shown in Table 1 (2) is shown in Figure 10 (c).
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<table>
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<td>Δd (s)</td>
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</tr>
<tr>
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</tr>
<tr>
<td>final radius</td>
<td>48 (pixels)</td>
<td>58 (pixels)</td>
</tr>
<tr>
<td>final timestep</td>
<td>2800 (times)</td>
<td>2000 (times)</td>
</tr>
<tr>
<td>computational time</td>
<td>4 (minutes)</td>
<td>3 (minutes)</td>
</tr>
</tbody>
</table>

Table 1: There are examples of properties to make dyeing stains on the dry and wet cloth and their data of the results. The result of properties (1) is Figure 10 (a) as on the dry cloth. The result of properties (2) is Figure 10 (c) as on the wet cloth. In this case, 2800 timesteps is worth 23 minutes at the actual time. Final timestep and computational time of (2) is up to the user.

Figure 4: Relationship between pressure and dyeing diffusion. (a) Dyeing table used as the set dyeing area. (b) Pressure table used as the set pressure area. We describe the dyeing system using these tables in Section 7. (c) is (b) after being Gauss filtered. (d) and (f) indicate the dispersion of dyeing and pressure. (f) shows the pressure with smoothing, and (d) shows the pressure without smoothing. In (d) and (f), red indicates the areas of maximum pressure, blue indicates the areas of medium pressure, and green indicates the areas being dyed. (e) is the resulting image of dye diffusion for (d), and similarly (g) is the resulting image of dye diffusion of (f).

5.3. Visualization of the Dye and Cloth

Basic color by amount of dye: Applying the algorithm from Section 5.2 produces the dispersion of the amount of dye. The algorithm can decide the basic color of the pixels. We assign the basic colors using the highest and lowest color values of a real image of dyeing with linear interpolation. In order to represent the dye, other color values can be added to determine the color from the real image of dyeing. The pigment change associated with dye stains appears in this way.

Cloth shading and fluff of fabrics: We present a simple method for the shading of dye on cloth. Figure 6 (a) is a magnified image of real dyeing and Figure 6 (b) is a contour drawing of the average value of RGB in Figure 6 (a). The part of this image corresponding to the cell can be represented with the hyperelliptic figures seen in Figure 6 (c). For each cell, we set the color distribution as in Figure 6 (c) based on the amount of dye. Figure 7 (a) is the image after applying the proposed dyeing model and Figure 7 (b) is the image after applying the proposed shading model.

Weaving yarns are made from many small pieces of fluff. The fluff of fabrics makes the outline of the threads look blurry and soft. In the proposed texture model, we use an image smoothing processing technique to make the contours of the weaving yarns vague to represent the texture of fluff, as shown in Figure 7 (c). By selecting weave patterns, we can produce various cloth textures, examples of which are shown in Figure 8. In addition, real cloth images can be used as the texture with the proposed model. In this case, it is necessary to indicate the weft and warp areas using another image.

6. Results

Figure 9 shows a comparison of real and computer-generated stains on dry cloth with the proposed cloth shading model. These images in Figure 9 show the results according to computer-generated characteristics of dyeing as Figure 2 in Section 4. The color distributions for of the each stain types are similar and both have a mottled appearance at the fringe. However, the dark edge and uneven arrangement of threads in the real stain are not achieved in our results and we will attempt to resolve this in future. Figure 9 (a) is a comparison between real and generated stains for the directional characteristics with an amount of dye. The outlines of the stains are similar for each amount of dye, suggesting that the proposed dyeing diffusion model is effective. Figures 9 (b) and 9 (c) show the characteristics associated with mottles and texture of the cloth. The proposed dyeing model with cloth patterns...
is effective for making mottles and cloth textures. Figure 9 (d) shows the results associated with dry pressure distribution; there was color gradation of dye for pressure and diffusion. Figure 10 shows comparisons of real and computer-generated stains on dry and wet cloth with a real cloth image.

7. Program Usage

In this section, we describe the process associated with generating dyeing images with the methods presented above.

**Dyeing table:** We can use an image in the proposed dyeing model to indicate the location of a diffusion area. Such an image is referred to as a “dyeing table”, which is used to generate dyeing-like images. Figure 11 (c) is created using the dyeing table. First, we draw patterns such as Figure 11 (b) by referring to the real tie-dyeing image Figure 11 (a). Then, the RGB values of Figure 11 (b) are converted to the amount of dye needed to generate the dyeing-like pattern Figure 11 (c).

**Pressure table:** Figure 9 (d) is the result of simulating dyeing with a pressure table that is a left image of Figure 12. The dye is distributed on four side of the cloth. In this way, we present a dyeing system that generates patterns with the protection against dyeing as indicated by the pressure table.

**Simple Tie-dyeing Simulation:** When the user wants a specific dyeing and pressure distribution, they can construct dyeing and pressure tables and produce the tie-dyeing image with the proposed system if it is possible to predict the tables. Figure 13 is an example of this process using a traditional Japanese pattern called seikaiha. Figure 14 shows the simulation of seikaiha along the time step. However, this example is simple, and it is easy to predict dyeing and pressure distribution. However, numerous dyeing techniques are difficult to predict and systems that can be used to predict the outcome of a dyeing pattern using either dyeing and pres-
sure tables from the geometry of the cloth, or from the dyeing techniques that were employed during dying is our future work.

8. Conclusion and Future Work

We have presented the dyeing model with an improved diffusion equation that can be used to consider weaving patterns, uneven diffusion coefficients, and different pressures for a tie-dyeing simulation. In addition, the improved diffusion equation presented herein cannot handle a wide range of diffusion coefficients because it is not robust. Consequently, the proposed model could be improved with more physical details such as gaps between the yarns. The extent of coloring calculated from the dye amount could also be defined physically with a suitable color coordinate system. It is difficult to consider the appearances of gaps or the details of cloth texture associated with dyeing in the present system. These physical improvements would simplify the decisions required by the user when using the proposed dyeing system. In addition, the proposed dyeing model should be physically justified. Additional future improvements could include increasing the speed of the proposed dyeing model and assigning the resolution of the cells according to level of detail of the cloth, adding mixed colors to the model, and so on. The rendering of the cloth could be improved by modeling the light interaction with the threads and 3D cloth geometry. Further research must be conducted to evolve the proposed model so that it can interactively handle three-dimensional configurations for the art of patterning cloth by dyeing. Adding dyeing techniques such as batik and brushing could also improve the completeness of the proposed dyeing system.

References


