2024 Master's Program, Graduate School of Design (General Entrance Examination) Achivement Test
Question Sheets

Mathematics, Biology, and Computer Science

Make sure that you are given 10 question papers and 5 answer sheets.

One answer sheet should be used per question.

Question [1] is compulsory. You must answer this question. (60 points).

Questions [2] - [9] are elective. Select and answer four questions from [2] through [9] (35 points each).

[1]

Select four from the following 20 words and describe the definition and characteristics of each.

(1) Codon
(2) Characteristics of Eukaryotes
(3) SNPs
(4) Epigenetics
(5) Enzymes
(6) Nucleotides
(7) Transcriptional regulators
(8) RNA processing
(9) Bijection
(10) Correlation coefficient
(11) Rank of a matrix
(12) Cumulative (sequential) integration
(13) Poisson distribution
(14) Jacobi matrix
(15) Recursive function
(16) Neural network
(17) Temporary and real arguments
(18) Bipartite graph
(19) Adjacency matrix
(20) Depth-first search

[2]

Answer the following questions regarding the function,

$$f(x) = \frac{1}{1+e^{-x}},$$

$$g(x) = \sin(\arctan x),$$

where arctan represents the inverse function of tan and the range of arctan is restricted to the principle value of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(1) Draw an outline of the graph of $f(x)$.

(2) Show $g(x) = \dfrac{x}{\sqrt{1+x^2}}$, taking into account that $\tan(\arctan x) = x$. In addition, draw an outline of the graph of $g(x)$.

(3) Find the value of the improper integral, $\displaystyle\int_{-\infty}^{0} f(x)dx$.

[3]

Let n be a positive integer, p, q, and r be real numbers that are $0 \leq p, q, r \leq 1$, and also $q + r \leq 1$. Let $\mathbf{A}$ be a 5×5 matrix below:

$$\mathbf{A} = \begin{pmatrix} 1-p & 0 & 0 & 0 & 0 \\ p & 1-p & q & 0 & 0 \\ 0 & p & 1-q-r & p & 0 \\ 0 & 0 & r & 1-p & p \\ 0 & 0 & 0 & 0 & 1-p \end{pmatrix}.$$

(1) Find all eigenvalues of $\mathbf{A}$.

(2) Among the eigenvectors corresponding to eigenvalue 1, one of the solutions of (1) above, find the eigenvector for which the sum of each component is unity.

(3) The following operations are repeated for four coins with two sides.

Count the number of coins that are face up or face down, and do one of the following depending on the situation.

- When there are more face-up coins, turn one of the face-up coins face down.
- When there are more face-down coins, turn one of the face-down coins face up.
- When there are the same number of face-up and face-down coins (2 coins each), choose one of the four coins at random. If the chosen coin is face up, do nothing, but if it is face down, turn it face up.

Let $s_{i,n}$ be the probability that the number of face-up coins is i after n repetitions ($n \in \{1,2,\dots\}$; $i \in \{0,1,2,3,4\}$). If the values of p, q, and r are appropriately chosen, the following equality holds:

$$\begin{pmatrix} s_{0,n} \\ s_{1,n} \\ s_{2,n} \\ s_{3,n} \\ s_{4,n} \end{pmatrix} = \mathbf{A} \begin{pmatrix} s_{0,n-1} \\ s_{1,n-1} \\ s_{2,n-1} \\ s_{3,n-1} \\ s_{4,n-1} \end{pmatrix} = \cdots = \mathbf{A}^n \begin{pmatrix} s_{0,0} \\ s_{1,0} \\ s_{2,0} \\ s_{3,0} \\ s_{4,0} \end{pmatrix},$$

where the vector $(s_{0,0} \quad s_{1,0} \quad s_{2,0} \quad s_{3,0} \quad s_{4,0})^{\mathrm{T}}$ represents the initial state of the four coins; if initially m coins are face-up, $s_{m,0} = 1$ and other components are zero. Find the values of p, q, and r for which the above equation holds.

(4) In (3), $s_{i,n}$ converges to a constant value as the number of iterations n increases. Find the convergence value of each probability.

[4]

Read the following text on the characteristics of living organisms and answer each question.

The three conventional features that distinguish living and non-living organisms are: compartment from the outer world, self-replication, and metabolism.

The compartment from the outer world is implemented by the structure of cells. The [(i)] divides the organism into an inner and an outer compartment. The [(i)] is composed of a [(ii)] bilayer membrane. Ionic concentrations inside and outside the cell often differ due to the compartmentalization. The resulting potential difference between inside and outside the cell is called the [(iii)]. Neurons in animals process and transmit information by changing the [(iii)].

Self-replication is implimented by cell division, i.e., one cell divides into two [(iv)] cells. (A)DNA replication takes place during the DNA replication phase ([(v)] phase), which precedes the cell division phase (M phase). The M phase is followed by the distribution of equal amounts of chromosomes to the [(iv)] cells. During DNA replication, the original DNA molecule is not maintained. Instead, semi-conservative replication takes place, i.e., the original DNA molecule is decomposed into two parts and the another new two molecules are synthesized based on the decomposed DNAs. In eukaryotes, (B)meiosis, a division with reduction of chromosomes, can occur in germates cells.

Metabolism is the set of chemical reactions that maintain biological activity. One purpose of matablism is to obtain energy, i.e., catabolism occurs to convert compounds into simpler substances. Conversely, the obtained energy can be used to synthesise other biomaterials, which is called [(vi)]. Aerobic respiration is one of the reaction in catabolism, consisting of glycolysis, [(vii)] and electron transfer. Aerobic respiration is mostly carried out in the cell organelle, [(viii)]. Most chemical reactions in metabolism are (c)enzymatic reactions. The basic metabolic pathways involved in maintaining life are referred to as primary metabolism and others as secondary metabolism.

(1) Write the most appropriate word for [(i)] to [(viii)] in the text.

(2) Explain the mechanism of DNA replication (underlined part (A)), based on the structure of DNA.

(3) The meiosis (underlined part (B)) contributes to the diversity of individuals within a species. Describe this mechanism.

(4) In relation to the enzymatic reaction in the underlined part (C), explain what role enzymes take?

(5) When constructing an evolutionary phylogenetic tree, which genes should be more focused, the genes in primary matabolism or secondary metabolism? Answer together with reasons.

(6) R. Dawkins pointed out that information transfer by self-replication occurs not only through the DNA level but also at the cultural level. Explain this idea.

[5]

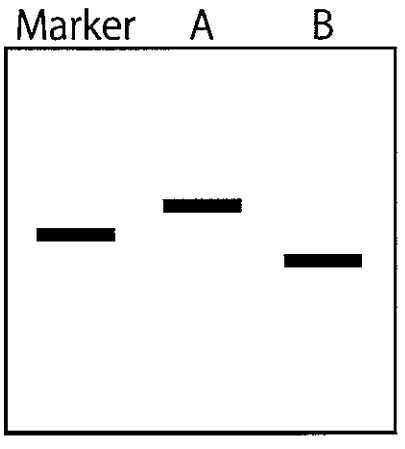
The nucleic acid sequence below is a region of a gene called *rbcL* in the chloroplast DNA of Arabidopsis. Answer the following questions regarding this sequence.

```
5' -  ATGTCGAGTA  GACCTTGTTG  TTTTGTTTTA  TTGCAAGAAT  TCTAAATTCA  TGACTTGTAG
      GGAGGGACTT  ATGTCACCAC  AAACAGAGAC  TAAAGCAAGT  GTTGGGTTCA  AAGCTGGTGT
      TAAAGAGTAT  AAATTGACTT  ACTATACTCC  TGAATATGAA  ACCAAGGATA  CTGATATCTT
      GGCAGCATTC  CGAGTAACTC  CTCAACCTGG  AGTTCCACCT  GAAGAAGCAG  GGGCTGCGGT
      AGCTGCTGAA  TCTTCTACTG  GTACATGGAC  AACTGTGTGG  ACCGATGGGC  TTACCAGCCT
      TGATCGTTAC  AAAGGACGAT  GCTACCACAT  CGAGCCCGTT  CCAGGAGAAG  AAACTCAATT
      TATTGCGTAT  GTAGCTTATC  CCTTAGACCT  TTTTGAAGAA  GGTTCGGTTA  CTAACATGTT
      TACCTCGATT  GTGGGTAATG  TATTTGGGTT  CAAAGCCCTG  GCTGCTCTAC  GTCTAGAGGA
      TCTGCGAATC  CCTCCTGCTT  ATACTAAAAC  TTTCCAAGGA  CCACCTCATG  GTATCCAAGT
      TGAAAGAGAT  AAATTGAACA  AGTATGGACG  TCCCCTATTA  GGATGTACTA  TTAAACCAAA
      ATTGGGGTTA  TCCGCTAAAA  ACTATGGTAG  AGCAGTTTAT  GAATGTCTAC  GTGGTGGACT
      TGATTTTACC  AAAGATGATG  AGAATGTGAA  CTCCCAACCA  TTTATGCGTT  GGAGAGACCG  - 3'
```

(1) A part of this region was PCR amplified by using the following primer set. Answer the expected length of the PCR product.

```
Forward:  5' -ATGTCACCACAAACAGAGACTAAAGC- 3'
Reverse:  5' -GTAAAATCAAGTCCACCACG- 3'
```

(2) PCR amplification in (1) was successful. The following electrophoretic image was obtained after the PCR product was electrophoresed using agarose gel with another PCR product and a DNA sized marker whose length is known to be 500 bp. Black band regions indicate the existence of DNA. Answer which of lanes A and B contains the PCR product amplified in (1), and explain the reason of your answer as well. During the electrophoresis, the upper side where the lane names are shown had a negative polarity and the lower side had a positive polarity.



(3) The following alignment shows a part of translated amino acid sequences of *rbcL* of four plant species. The position of each amino acid site corresponds between sequences, and there are no gaps in the alignment. Explain procedures to create a phylogenetic tree using these four amino acid sequences, and draw the phylogenetic tree. Line lengths need not reflect the accurate distance, and lines are not necessarily drawn to be straight.

```
Arabidopsis :  HIEPVPGEETQFIA
     Tomato :  RIERVVGEKDQYIA
     Cotton :  DIEPVPGEEDQYIC
     Ginkgo :  DIEPVPGEENQFIA
```

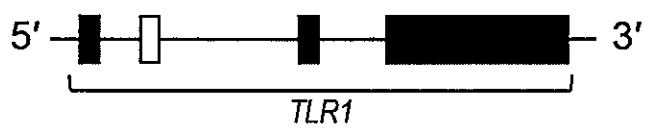
[6]

Antient DNA analyses have revealed that several human lineages such as Neanderthal and Denisovan were living until several ten thousands years ago. Answer the following questions about researches on human evolution.

(1) Both mitochondrial DNA and nucleic DNA have been used for phylogenetic analysis using antient DNA. Explain the advantages and disadvantages of each material for the analysis.

(2) It has been revealed that a few percent of genomic information of Neandertal and Denisovan is included in genomes of some modern human populations which had expanded into Eurasia or Melanesia. Describe what you can conjecture on the relationships between Neandertal and Denisovan, and modern human populations.

(3) The below schematic shows the structure of the gene *TLR1*, which encodes a Toll-like receptor that acts as a sensor to identify characteristic structures of bacteria and viruses in the innate immune system.



The boxes indicate exons. The exon depicted as a white box can be included or not included in the mature mRNA depending on the differences in single nucleotide polymorphisms. According to a research, this exon tends to be included in the mature mRNA when a certain nucleotide position in the white box is C, which is considered to have been popular in Neandertals, but not when this position is G, which is popular in modern human populations. Note that whether this region is likely to be included in the mature mRNA or not also depends on the organ where the gene is expressed.

(a) What is the gene region between exons, which is excluded from mature mRNAs, called?

(b) Explain the benefits of the mechanism that produces different mature mRNA sequences from the same DNA sequence of a genome.

(4) The process of the human evolution has been revealed using the DNA extracted from antient human bones excavated mainly in northern Eurasia. Describe your view on the remaining issues in understanding the overall picture of human evolution.

[7]

For each of the following questions, write your discussion. In marking the answer, logicality of argument will be evaluated on.

(1) Explain in 100 words or less the symbiosis between individuals of different species, distinguishing it from non-symbiosis coexistence.

(2) The amount of seeds that fell to the ground from a parent tree was greater the closer the tree was to the ground (Fig. 1). The number of offspring that subsequently grew to a certain developmental stage was distributed with peaks at some distance from the parent (Fig. 2).

(a) Based on the above observations, describe the relationship between the probability of a seed growing to that developmental stage and its distance from the parent tree. You may draw a diagram plot.

(b) Write a discussion on the causes of a. You may write a discussion focusing on the interactions between this tree species and its predators and pathogens.

(c) Write a discussion on the reasons why tree species diversity is maintained in natural forests.

Fig. 1

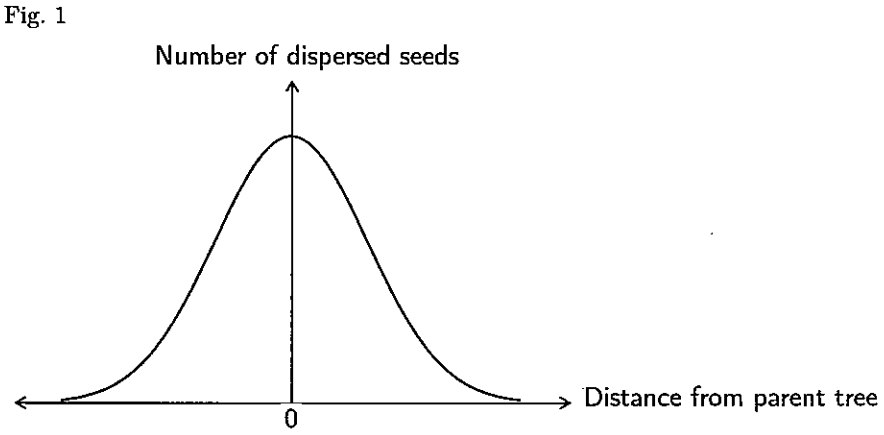
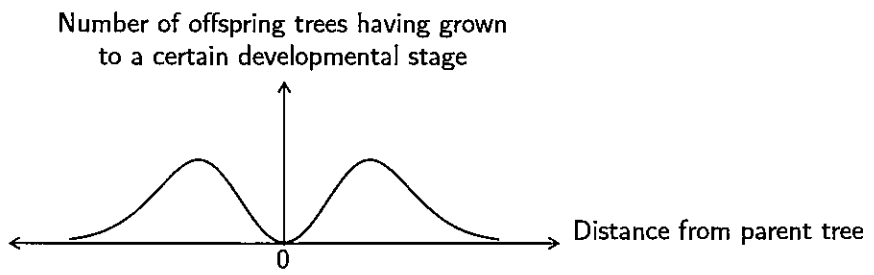


Fig. 2



[8]

Suppose that one rope of length L cm is cut into several pieces, and a rope of length ℓ cm is sold at a price of v_ℓ yen as shown in the table.

ℓ	1	2	3	4	5	6	7	8	9	≥ 10
v_ℓ	1	4	9	10	12	13	14	15	16	17

Also suppose that a rope length (cm) is always an integer. In this case, let P_L be the maximum selling price of the rope of length L.

(1) Enumerate all combinations of lengths of a rope with $L = 4$, and also find the total price of each case.

(2) For $\ell = 1, \ldots, L$, find the recursion formula of P_ℓ using v_ℓ, $P_m (1 \leq m \leq \ell - 1)$, max (function returning the maximum among arguments).

(3) Describe a computer program finding P_L using the recursion formula in the previous question. You are allowed to use, as a representation of computer programs, a natural language, mathematical equations, and pseudo-programming languages.

[9]

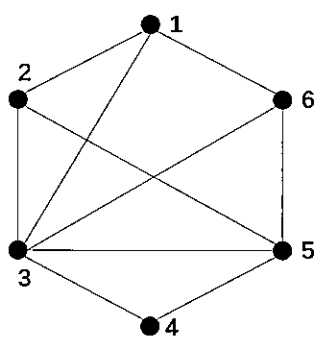
Graph $G = (V, E)$ is an undirected graph composed of a set $V = \{v_1, v_2, \ldots, v_n\}$ consisting of n nodes and a set E of edges consisting of 2 nodes.

Note that it does not include edges that connect the same node (self-loops) and multiedges (that is, any pair of two nodes has at most one edge).

When v_i and v_j are joined by an edge, it is described that node v_i and v_j are adjacent to one another.

If v_i and v_j are adjacent to one another, $a_{ij} = 1$; otherwise, the $n \times n$ matrix A_G, whose (i, j) element is the variable a_{ij} with $a_{ij} = 0$, is called the adjacency matrix of graph G.

We also say that a sequence of adjacent nodes is called a path of the graph. When the path consists of m nodes, the length of the path is $m - 1$. For example, in the graph H, the sequence of nodes, $1, 2, 5, 6, 1, 2, 5$, is a path of length 6.



H

Answer the following questions.

(1) A complete graph is a undirected graph in which every pair of distinct node is connected by an edge. We denote the complete graph of n nodes by K_n. Find the total number of edges of K_n using n.

(2) Find the adjacency matrix of graph H.

(3) In graph G, let the total number of paths of length k from vertex i to j is denoted by $P_G(i, j, k)$.

(a) Calculate $P_H(3, 3, 3)$.

(b) We denote the k-th power of A by A^k, and the (i, j)-element of A_G^k by $a_{ij}^{(k)}$. Prove $a_{ij}^{(k)} = P_G(i, j, k)$.