2023 Master's Program, Graduate School of Design (General Entrance Examination) Achivement Test
Question Sheets

Mathematics, Biology, and Computer Science

Make sure that you are given 10 question papers (including this page) and 5 answer sheets.

One answer sheet should be used per question.

Question [1] is compulsory. You must answer this question. (60 points).

Questions [2] - [9] are elective. Select and answer four questions from among [2] through [9] (35 points each).

Mathematics, Biology, and Computer Science

[1]

Select four from the following 20 words and describe the definition and characteristics of each.

(1) Cellular membrane
(2) Genome
(3) Semiconservative replication
(4) Electrophoresis
(5) Cellular respiration
(6) Restriction enzyme
(7) Retrotransposon
(8) Life cycle
(9) Jacobian matrix
(10) Continuous function
(11) Type I error and Type II error in statistics
(12) Implicit function
(13) L'Hôpital's rule
(14) Bijection
(15) Newton's method
(16) Notation system of base n
(17) Euler method
(18) Tree in graph theory
(19) Recursive function
(20) Dynamic programming

[2]

Answer the following questions regarding the function,

$$y = \tan^{-1} x,$$

where $\tan^{-1}$ is the inverse function of tan and the range of $\tan^{-1}$ is restricted to the principle value of $(-\frac{\pi}{2}, \frac{\pi}{2})$.

(1) Draw the graph, $y = \tan^{-1} x$. Is this function even or odd?

(2) Find the value of $\tan^{-1} \dfrac{1}{\sqrt{3}} + \tan^{-1} \sqrt{3}$.

(3) Let $a = \tan^{-1} \dfrac{1}{3}$, $b = \tan^{-1} \dfrac{1}{2}$.

Find the value of $\tan(a+b)$. In addition, find the value of $a + b = \tan^{-1} \dfrac{1}{3} + \tan^{-1} \dfrac{1}{2}$.

(4) Show $\dfrac{dy}{dx} = \dfrac{1}{x^2+1}$.

(5) Based on the result of (3) and (4), calculate the value of $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{3}} \frac{1}{x^2+1} dx$.

[3]

Let n be a positive integer. Let b, x_0, and y_0 be positive real numbers. Let $\mathbf{A}$ and $\mathbf{B}$ be the following square matrix of order 2.

$$\mathbf{A} = \begin{pmatrix} 0 & 5/4 \\ 1/2 & 3/4 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & b \\ 1/2 & 3/4 \end{pmatrix}$$

(1) Show by calculation that the two eigenvalues of $\mathbf{A}$ are $5/4$ and $-1/2$.

(2) For each of the above eigenvalues, obtain the corresponding eigenvectors and choose and show one that has only integer components.

(3) Find $\mathbf{A}^n$ and fill ① to ⑧ below. The calculation process must also be shown.

$$\mathbf{A}^n = \frac{1}{7} \cdot \begin{pmatrix} ① \cdot \left(\frac{5}{4}\right)^n + ② \cdot \left(-\frac{1}{2}\right)^n & ③ \cdot \left(\frac{5}{4}\right)^n + ④ \cdot \left(-\frac{1}{2}\right)^n \\ ⑤ \cdot \left(\frac{5}{4}\right)^n + ⑥ \cdot \left(-\frac{1}{2}\right)^n & ⑦ \cdot \left(\frac{5}{4}\right)^n + ⑧ \cdot \left(-\frac{1}{2}\right)^n \end{pmatrix}$$

(4) Let $\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \mathbf{A}^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$. Show that the sequences $\{x_n\}$ and $\{y_n\}$ are divergent while $\{y_n/x_n\}$ is convergent. Then, find the limit of $\{y_n/x_n\}$.

(5) Find the condition for the positive real number b to satisfy $\lim_{n\to\infty} \mathbf{B}^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

[4]

Read the following text on biodiversity and answer each question.

Biodiversity consists of "ecosystem diversity", "species diversity", and "genetic variability".

Ecosystem diversity refers to the existence of ecosystems that depend on climate and topography, such as lakes, rivers, oceans, coral reefs, forests, and grasslands. The functions of various ecosystems are integrated to maintain the local or global environment.

Species diversity refers to the presence of a variety of species, from plants and animals to microorganisms, in a specific area. Several classification methods for species have been proposed so far. Handreds years ago, in natural history, organisms were classified into (A)animal and plant kingdoms. In 20th century, Chatton focused on cellular structure, and proposed the two classifications of (B)cellular structure: [(i)] without a nucleus and [(ii)] with a nucleus. More recently, C. Woese proposed the idea of domains and further classified organisms into three categories based on their genome sequences: [(ii)] , [(iii)] , and [(iv)] . This classification is now widly accepted. (C)The evolutionary phylogenetic tree is an effective visualication for relations and diversity of the species.

On the other hand, some properites are universal among species. For example, the process of mRNA synthesis based on DNA sequences, called [(v)] , and the process of protein production based on mRNA sequences, called [(vi)] , are common to all living organisms. [(vi)] proceeds on a complex molecular apparatus, called [(vii)] , resulting in polypeptide chains. [(vii)] reads [(viii), number] nucleotides of mRNA at a time and polymerise a specific amino acid in protein. This DNA base-amino acid correspondence, which is called the [(ix)] table, is common across species.

Genetic variability refers to the diversity of genes within a species, which is caused by changes in DNA sequences. For example, a single nucleotide mutation with a frequency of more than 1 percent, can result in polymorphism in a population, which is called [(x)] . For example, the diversity of alcohol metabolism among the Japanese is attributed to a polymorphism in the ALDH2 gene.

(1) Write the most appropriate words for [(i)] to [(x)] in the text.

(2) Name one organism that has the characteristics of both the animal kingdom and the plant kingdom in the classification in natural history (underlined part (A)).

(3) Describe a different point among the two classifications in the underlined part (B) except for the existance of a nucleus.

(4) Explain briefly how to construct an evolutionary phylogenetic tree (the underlined part (C)).

(5) Recently, it has been focused that diversity in DNA sequences is not the only factor responsible for phenotypic diversity. Describe another mechanism responsible for this.

[5]

Answer the following questions about genetic information in organisms.

(1) Adenine (A) accounts for 30% of the bases in human genomic DNA when calculated by the number of molecules. Answer the proportions of other bases.

(2) Explain the differences in molecular structure between DNA and RNA.

(3) Describe how the differences in molecular structure between DNA and RNA that you have answered in (2) relate to the functions of DNA and RNA in vivo. If there are multiple functions, answer more than one function.

(4) If homologous regions of genomes are compared between generations or between species, some regions exhibit more variations than other regions. Describe your idea on why this phenomenon is observed.

[6]

Answer the following questions about biotechnology.

(1) Explain in detail the principle of the polymerase chain reaction (PCR).

(2) Describe the characteristics of genome editing techniques such as CRISPR-Cas9 as compared to conventional gene recombination techniques.

(3) Give an example of biotechnology that you know about, and explain the details of that technology. Then, describe your idea on how the development of the technology may affect the organisms in the future.

[7]

Answer each of the following questions. When marking, the correctness of logical development will be evaluated.

(1) There is a trend along latitude in body-size variation of mammalian related species: For example, polar bears are larger than sun bears that inhibit in low-latitude regions. Discuss from the point of view of adaptative evolution the reason why such a body-size variation occurs along latitude.

(2) There are opposite trends in body-size evolution in island vertebrates: small animals tend to be greater and large animals tend to be smaller in island species than continental related species. Provide two or more conditions that are specific in island environments and are thought to have contributed to these trends, and discuss those conditions from the viewpoint of adaptative evolution.

(3) Evolution from herbaceous plants to woody plants and that from woody to herbaceous plants are known to have occurred relatively frequently. Which one of the above two evolutionary changes is more likely to occur in islands? Give discussion on the reason for your answer.

[8]

Answer each of the following questions:

(1) A palindrome is a string of characters, which reads the same backword as forward, such as radar. Suppose a string of length L is stored in array A. Write an algorithm to determine whether the string is a palindrome or not, where A[i] $(i = 0, 1, \ldots, L - 1)$ is the ith character of the string.

(2) Consider a string on uppercase and lowercase alphabetic characters, A, ..., Z, a, ...,z. We denote $X \sim Y$ if two strings X and Y match, ignoring case differences.

(a) For a string S of length L, answer the number of strings X satisfying $S \sim X$.

(b) Write an algorithm that prints all strings X that satisfy $S \sim X$ for any string S of length 1 or more. Note that you are allowed to use function, "upper," which returns the uppercase letter of an input character, ℓ, and function, "lower," which returns the lowercase letter of ℓ.

[9]

Below is a pseudocode for the depth-first search algorithm for an undirected graph $G = (V, E)$:

```
function DepthFirstSearch(v):
    label vertex v as visited
    print v
    for each u adjacent to v
    /* If there are multiple candidates of v, the smaller one will be chosen. */
        if u is not visited
            DepthFirstSearch(u)
```

(1) Let $G = (V, E)$ be an undirected graph where $V = \{0, 1, 2, 3, 4, 5\}$, $E = \{\{0, 1\}, \{1, 3\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 5\}\}$. When a vertex $r = 0$, show the vertices to be printed while DepthFirstSearch(r) is being executed in order.

(2) Show the worst-case time complexity of DepthFirstSearch for an undirected graph $G = (V, E)$, using the size of V, $|V|$, and the size of E, $|E|$.