Acoustic Engineering / Signal Processing

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question I (40 points)

Consider a single-degree-of-freedom mass-spring-damper system shown in the figure below. Its damping is light and a driving force is applied. Its governing equation is given by

$$M\frac{d^2x}{dt^2} + R\frac{dx}{dt} + Kx = f,$$
$$f = F\cos(\omega t) = \mathrm{Re}\left[Fe^{j\omega t}\right],$$

where x is the displacement of the mass, M the mass, R the resistance of the damper, K the stiffness of the spring, f the driving force, F the driving force amplitude, ω the angular frequency of the driving force, t the time, and j is an imaginary unit. Re[ ] is the operator to extract the real part. In addition, $\omega_0 = \sqrt{K/M}$, $h = R/M$. Answer the following questions.

(1) Suppose that the stationary solution of x is expressed in terms of the complex amplitude D as follows:

$$x = \mathrm{Re}\left[De^{j\omega t}\right].$$

Write down the expression of D, using necessary ones from M, R, K, F, ω, j, ω_0, and h. (5 points)

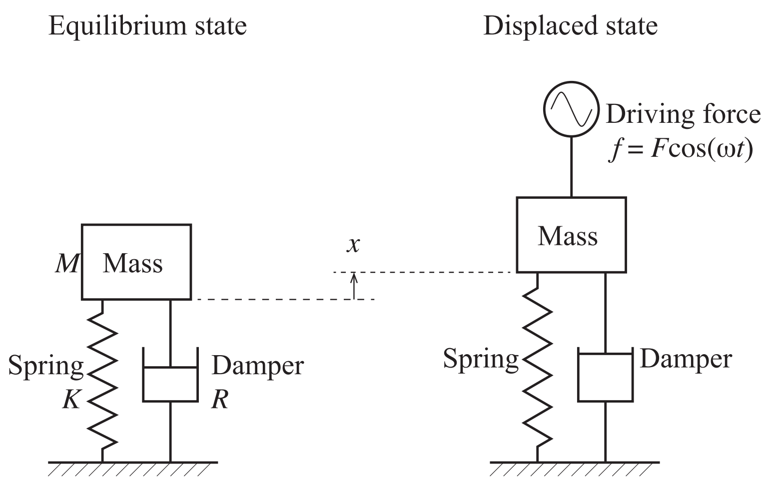
(2) Suppose that the stationary solution of x is expressed in terms of only the real number as follows:

$$x = A\cos(\omega t + \phi).$$

Write down the expressions of A, $\cos\phi$, and $\sin\phi$, using necessary ones from M, R, K, F, ω, j, ω_0, and h. (5 points)

(3) Draw a rough outline of a graph of the amplitude-frequency response of this system (with angular frequency ω on the horizontal axis and amplitude A or absolute value of complex amplitude D on the vertical axis), and that of a graph of the phase-frequency response (with angular frequency ω on the horizontal axis and phase ϕ on the vertical axis). Explicitly give the values at $\omega = 0$, $\omega = \omega_0$, and $\omega \to \infty$, using necessary ones from M, R, K, F, j, ω_0, h, and π. (7 points)

(4) Show the angular frequency $\omega_{\max}$ of the driving force when the amplitude of the stationary solution of x is strictly maximum, and the maximum value $A_{\max}$ of the amplitude at that angular frequency, using necessary ones from M, R, K, F, j, ω_0, and h. (7 points)



Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question I (Continued) (40 points)

Next, consider a single-degree-of-freedom mass-spring-damper system shown in the figure below. Its damping is light and the base follows a single harmonic motion of the angular frequency ω. Its governing equation is given by

$$M\frac{d^2x_2}{dt^2} + R\left(\frac{dx_2}{dt} - \frac{dx_1}{dt}\right) + K\left(x_2 - x_1\right) = 0,$$
$$x_1 = A_1\cos\left(\omega t\right) = \mathrm{Re}\left[A_1 e^{j\omega t}\right],$$

where x_1 is the displacement of the base, x_2 the displacement of the mass, M the mass, R the resistance of the damper, K the stiffness of the spring, and A_1 is the amplitude of x_1. In addition, $\omega_0 = \sqrt{K/M}$, $h = R/M$. Answer the following questions.

(5) Suppose that the stationary solution of x_2 is expressed in terms of the complex amplitude D_2 as follows:

$$x_2 = \mathrm{Re}\left[D_2 e^{j\omega t}\right].$$

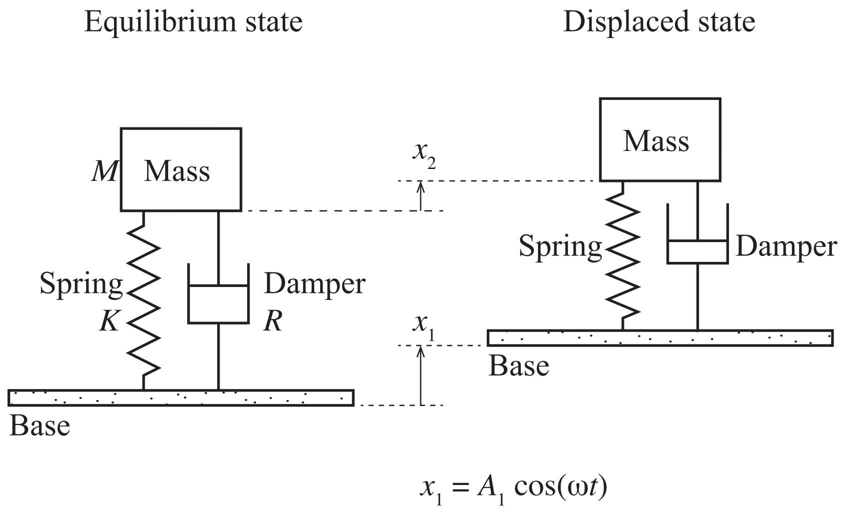
Write down the expression of D_2, using necessary ones form M, R, K, A_1, ω, j, ω_0, and h. (5 points)

(6) Suppose that the stationary solution of relative displacement $y = x_2 - x_1$ with respect to the base is expressed in terms of the complex amplitude G as follows:

$$y = \mathrm{Re}\left[G e^{j\omega t}\right].$$

Write down the expression of G, using necessary ones from M, R, K, A_1, ω, j, ω_0, and h. (5 points)

(7) Draw a rough outline of a graph of the amplitude-frequency response of the relative displacement y (with angular frequency ω on the horizontal axis and absolute value of complex amplitude G on the vertical axis). Explicitly give the values at $\omega = 0$, $\omega = \omega_0$, and $\omega \to \infty$, using necessary ones from M, R, K, A_1, j, ω_0, h, and π. (6 points)



Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question II (40 points)

A sinusoidal plane sound wave is propagating along the x-axis in free space, where the instantaneous value of the velocity potential is expressed as

$$\phi(x,t) = \mathrm{Re}\left[A\, e^{j(\omega t - kx)}\right]$$

Here, Re denote the real part of a complex number is to be found, A the real amplitude, ω the angular frequency, c the speed of sound, $k = \omega/c$ the wavenumber, ρ_0 the density of the medium and t the time. Also $j = \sqrt{-1}$ (imaginary unit). Answer the following questions.

(a) Show the units of the A in the SI units. (8 points)

(b) Show the sound pressure and particle velocity of this plane wave. (8 points)

(c) The sound intensity of this plane wave can be derived by averaging the product of real part of the above sound pressure and particle velocity over time (period). Show that this value is $\rho_0 \omega k A^2/2$. (8 points)

(d) Consider the level notation of the physical quantity produced by this plane wave.

(d-1) Show the sound pressure level and sound intensity level of this plane wave using the symbols obtained in the above equation. (4 points)

(d-2) For plane waves, the sound pressure level and the sound intensity level are said to be equal, but this is based on an approximation (rounding of the numbers). Explain what kind of approximation this is. Also indicate whether the approximation is more likely to hold when the temperature is low or high. (4 points)

(e) If there exists a plane wave on the same x-axis whose velocity potential is described by $\phi'(x,t) = \mathrm{Re}\left[B\, e^{j(\omega t + kx)}\right]$ with B being the real amplitude, what is the maximum dB increase in sound pressure level observed on the x-axis? Show the equation together with the derivation process. (8 points)

Acoustic Engineering / Signal Processing

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question III (40 points)

Answer each of the following questions regarding a discrete-time system $H(z)$ where the input-output relationship of the system is represented by the following difference equation.

$$y(n) = x(n) - x(n-2)$$

The z-transformation of the sequence $x(n)$ is $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$. Also j is an imaginary unit.

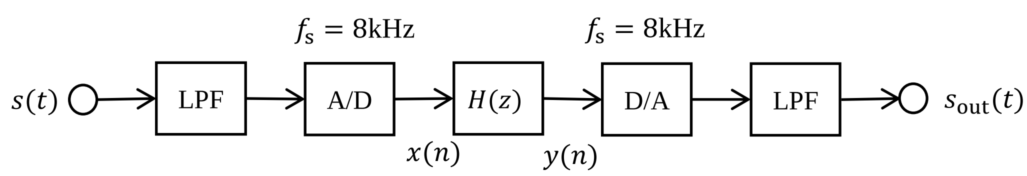
(1) Show the transfer function $H(z)$ of this system. (10points)

(2) Draw the zero point and the pole as $\circ$ and $\times$ respectively on the z-plane. (5points)

(3) Show the frequency response $H(e^{j\omega})$ of this system $H(z)$. Furthermore, draw the amplitude frequency response $|H(e^{j\Omega})|$ and the phase frequency response $\angle H(e^{j\Omega})$, respectively. The horizontal axis Ω should be displayed in the range $-\pi \leq \Omega < \pi$. (10points)

(4) Consider the signal processing system in the figure below, which combines a system $H(z)$, a low-pass filter (LPF), and A/D and D/A conversion systems with sampling frequency $f_s = 8$ kHz. An arbitrary continuous signal $s(t)$ was input to this signal processing system and a continuous signal $s_{out}(t)$ was obtained as output. When an arbitrary continuous signal $s(t)$ is input to this signal processing system, the output is the continuous signal $s_{out}(t)$. Show the conditions required for a low-pass filter (LPF) in this signal processing system. An ideal LPF is used for the LPF, and the number of quantization bits is sufficiently large that quantization distortion is negligible. (10points)

(5) In the above question (3), what spectral deformation does the output signal $s_{\mathrm{out}}(t)$ for any continuous signal $s(t)$ input undergo? Explain using specific frequencies, etc. Note that the spectrum is assumed to use an continuous-time frequency analysis. (5points)



The signal processing system

Acoustic Engineering / Signal Processing

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question IV (40 points)

Answer the following questions for the moving coil type electro-dynamic transducer.
Note that, the imaginary unit shall be written as j.
When performing numerical calculations, the significant digits shall be three digits, and the π shall be 3.14.

(1) Let the magnetic flux density be B [T] and the length of the wire that makes up the coil be ℓ [m]. When a current I [A] is applied to this coil, find the electromagnetic force F' [N] acting on the coil. (5 points)

(2) In question (1), $B = 1.2$ T, $\ell = 3.2$ m, and $I = 0.5$ A. Find the electromagnetic force F' acting on the coil. Indicate the units. (5 points)j

(3) Assume that the total mass of the oscillating part of the electro-dynamic transducer in question (2) is $m = 20$ g. When a sinusoidal current of frequency 200 Hz and effective value I = 1.0 A is applied to the coil, find the effective value V of the velocity at which the vibrating system moves. Indicate the units.

Note that this frequency is in the mass control region. Also, assume that the stiffness of the springs supporting the vibrating system is negligible and that there is no mechanical resistance. (10 points)

(4) Under the conditions of Question (3), find the effective value of the vibration displacement amplitude X of the vibration system? Indicate the units. (5 points)

(5) Find the dynamic impedance $Z_{em} = R_{em} + j\, X_{em}$ of the electrodynamic transducer in question (3) viewed from the coil terminal at a frequency of 200 Hz. (10 points)

(6) The electrical impedance between the electrical terminals of a certain movable coil type electro-dynamic transducer, which is different from question (3), was $Ze = 10.2 + j\, 2.3$ [Ω] when it was measured at a certain frequency with the vibrating system in a state where it can vibrate. When the vibrating system was fixed so that it would not move and measured at the same frequency, $Z_{e0} = 10.0 + j\, 2.9$ [Ω]. What is the value of the dynamic impedance $Z_{em} = R_{em} + j\, X_{em}$ with this electro-dynamic transducer ready in a state where it can vibrate? Indicate the units. (5 points)

Acoustic Engineering / Signal Processing

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question V (40 points)

There is a room with a surface area of S [m^2] and a volume of V [m^3]. There is a point sound source with acoustic power W [W] in the room that radiates a spherical wave, but the sound field is diffuse outside its vicinity. In this case, answer the following questions. Here, the sound speed is c [m/s], the energy density in the room is E [J/m^3], and the average sound absorption coefficient in the room is $\overline{\alpha}$. Using the fact that the process of energy decay in a diffuse sound field is expressed as $V\dfrac{dE(t)}{dt} = -V\dfrac{E(t)}{\tau}$ and that the incident energy on a unit area of a boundary surface in contact with a diffuse sound field in unit time is given by $\dfrac{Ec}{4}$, answer the following questions.

(a) Define diffuse sound field and reverberation time. (7 points)

(b) Show the relationship between the variable τ and reverberation time in the equation for the energy decay process. (6 marks)

(c) Define the mean free path ℓ_m and show its value in this room. (6 points)

(d) In the room, the average sound pressure level at a distance from the sound source was L [dB]. Show the formula giving the power level PWL [dB] of the sound source, together with the derivation process. (7 points)

(e) On the other hand, if the sound pressure level near the sound source at a distance r [m] from the center of the sound source is $L'(r)$ [dB], the power level of the source is expressed as follows.

$$\mathrm{PWL} = L'(r) - 10\log_{10}\left(\frac{1}{4\pi r^2} + \frac{4}{R}\right)$$

Show how the room constant R is defined here, and also the physical meaning of the two terms in the brackets on the right-hand side.

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question V(Continued) (40 points)

(f) The reverberation time of this room when it was empty was T_0. In addition, the reverberation time was T_1 [s] when a material with a statistical sound absorption coefficient of 0.5 and an area of S_1 [m^2] was placed at the boundary of the room. Show that the area S_1 of the material can be expressed as follows by using Sabine's reverberation formula. (7 points)

$$S_1 = \frac{T_0 - T_1}{T_1} \frac{2S\overline{\alpha}}{1 - 2\overline{\alpha}}$$

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question VI (40 points)

The following equation is the difference equation representing a causal linear time-invariant system in discrete time.

$$y(n) = \frac{1}{8}\{x(n) + 3x(n-1) + 3x(n-2) + x(n-3)\}$$

Note that the Fourier transform of a discrete-time signal $x(n)$ is given as follows. Here, j is the imaginary unit.

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n}$$

(1) Find the impulse response $h(n)$ of the system. (6 points)

(2) Find the frequency response $H(\Omega)$ of the system. (6 points)

(3) Find the amplitude response $|H(\Omega)|$ and phase response $\angle H(\Omega)$ from the frequency response $H(\Omega)$. Note that $\angle H(\Omega)$ is a linear function of Ω and it has no discontinuities within the range $-\pi < \Omega \leq \pi$. (6 points)

(4) Draw an outline of the amplitude response $|H(\Omega)|$ for $0 \leq \Omega \leq \pi$ along with the values of $|H(0)|$ and $|H(\pi)|$. In addition, show that this system has the characteristic of a low-pass filter. (6 points)

(5) Draw the phase response $\angle H(\Omega)$ for $0 \leq \Omega \leq \pi$. (6 points)

(6) Let $x_1(n)(n = 0, 1, 2, 3, ..., N-1)$ be a signal of length N, and further define the signal $x_2(n) = (-1)^n x_1(n)$. Let $X_1(\Omega)$ be the Fourier transform of $x_1(n)$ and $X_2(\Omega)$ be the Fourier transform of $x_2(n)$. Then show that the following relation holds true. (5 points)

$$X_2(\Omega) = X_1(\Omega - \pi)$$

(7) When $h(n)$ is the impulse response obtained in the question (1) above, define the impulse response $g(n) = (-1)^n h(n)$. Draw an outline of the amplitude response $|G(\Omega)|$ for this new system, and show that it has the characteristic of a high-pass filter. (5 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question VII (40 points)

The Fourier transform $X(\omega)$ of a continuous-time signal $x(t)$ is given by the following equation. Here, j is the imaginary unit.

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

In addition, note that the polynomial expansion of a sinusoidal function is given by $\sin x = x - x^3/3! + x^5/5! - \cdots$.

(1) Let $X(\omega)$ be the Fourier transform of a signal $x(t)$ and $x'(t) = x(t - t_0)$ is the time-shifted signal of $x(t)$ with the shift width of t_0. Find the Fourier transform of $x'(t)$ as $X'(\omega)$ using $X(\omega)$. (6 points)

(2) Let $X(\omega)$ be the Fourier transform of a signal $x(t)$ and $X(\omega - \omega_0)$ is the frequency-shifted transform of $X(\omega)$ with the shift width of ω_0. Show that the inverse Fourier transform of $X(\omega - \omega_0)$ is $e^{j\omega_0 t} x(t)$. (6 points)

(3) Find the Fourier transform of $f(t) = \cos(\omega_0 t)$ as $F(\omega)$. Note that the Fourier transform of $x(t) = 1$ is $X(\omega) = 2\pi\delta(\omega)$, where $\delta(\omega)$ is the following delta function. (7 points)

$$\delta(\omega) = \begin{cases} 0, & \omega \neq 0 \\ \infty, & \omega = 0 \end{cases} \quad , \quad \int_{-\infty}^{\infty} \delta(\omega) d\omega = 1$$

(4) Find the Fourier transform of $g(t) = \cos^2(\omega_0 t)$ as $G(\omega)$. (7 points)

(5) Find the Fourier transform of the following rectangular signal $s(t)$ as $S(\omega)$. (7 points)

$$s(t) = \begin{cases} \dfrac{1}{T_0} & |t| \leq T_0/2 \\ 0 & |t| > T_0/2 \end{cases}$$

(6) Among the frequencies for which $S(\omega) = 0$, consider the minimum frequency within the range of $\omega > 0$ (ω_1) and the frequency with minimum absolute value within the range of $\omega < 0$ (ω_2). In addition, let the width of the main lobe of $S(\omega)$ be defined as $\Delta\omega = \omega_1 - \omega_2$, which is the difference between ω_1 and ω_2. In this case, show that $\Delta\omega$ is inversely proportional to the length of the rectangular signal (T_0). (7 points)

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing

Answer Sheets

Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing	Answer Sheets (page 11 of 11)	Examinee's number

Use a separate answer sheet for each question.

Do not write on the back side of the answer sheet, or your answers will not be marked.