Acoustic Engineering / Signal Processing

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question I (40 points)

Consider a single-degree-of-freedom vibration system with light damping. The vibrational displacement $x(t)$ when the vibration system oscillates freely is expressed by

$$x(t) = A\exp\left(-\frac{1}{2}kt\right)\cos(\omega_d t + \phi)$$

where t is a time variable, k is the damping coefficient, and ϕ is the initial phase. The symbol ω_d is the damped natural angular frequency given by

$$\omega_d = \sqrt{\omega_0^2 - \frac{k^2}{4}}$$

where ω_0 is the natural angular frequency without damping. The Q-value is defined by $Q = \omega_0/k$.

(1) We observe the waveform of vibrational displacement $x(t)$. From this, we readout the period T_d and the ratio r of any maximum of the vibrational displacement x_a to the next x_b :

$$r = \frac{x_a}{x_b}$$

The quantity $\log_e r$ is called the *logarithmic decrement*. Using the *logarithmic decrement* $\log_e r$, T_d, and the natural angular frequency without damping ω_0, we can identify the value of Q of the vibration system. Show Q, using $\log_e r$, T_d, and ω_0. (8 points)

(2) When $k \ll 1$, an approximate relation among ω_0, ω_d, and Q exists. Show the approximate relation. You may use the approximation $(1+d)^a \approx 1 + ad$, when $|d| \ll 1$. (8 points)

(3) Observing the waveform of vibrational displacement in damped oscillation, measuring the mass and stiffness, and using the approximate relation among ω_0, ω_d, and Q derived in (2), we can also identify the value of Q of the system. Explain the procedure and mathematical expressions for identifying the value of Q. (8 points)

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〔Compulsory Question〕

Question I(Continued) (40 points)

Consider a single-degree-of-freedom vibration system with light damping, which is driven by the external force of amplitude F and angular frequency ω.

(4) The amplitude X of the vibration displacement in a stationary state is expressed by

$$X = \frac{F}{M}\frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + k^2\omega^2}}$$

where M is mass. Show X for $\omega = 0$ and $\omega = \omega_0$ respectively, using the necessary ones from F, M, ω_0, ω, Q. Then, from this, explain what does the Q-value refer to in the amplitude of the vibration displacement in a stationary state. (8 points)

(5) The time-averaged power absorbed from the external force is given by

$$\overline{Pw} = \frac{F^2}{2R}\frac{k^2\omega^2}{(\omega_0^2 - \omega^2)^2 + k^2\omega^2}$$

where R is resistance coefficient. Based on the observation result of the frequency characteristics of $\overline{Pw}$ (the response curve of $\overline{Pw}$ when having ω change), we readout the value of ω giving the maximum of $\overline{Pw}$ and that giving the half of the maximum, thereby identifying the Q-value. Explain the procedure for identifying the Q-value and mathematical expressions used for the procedure. (8 points)

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〔Compulsory Question〕
Question II

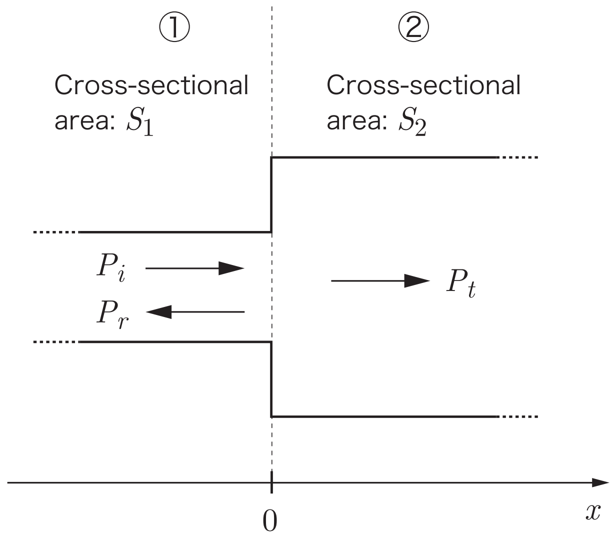
Answer the following questions about sound waves. Where ω is the angular frequency, t is the variable for time, k is the wavenumber, and x is the variable for position along the x axis.

(1) If there are two sound sources and each radiates sound independently at a certain position, the instantaneous sound pressure is expressed as follows.

$$p_1(t) = A_1 \sin(\omega t + \varphi_1), \quad p_2(t) = A_2 \sin(\omega t + \varphi_2)$$

where $A_1,\ A_2$ are real amplitudes and $\varphi_1,\ \varphi_2$ are initial phases. If two sources radiate sound simultaneously, express the sound pressure level observed at the same location using $A_1,\ A_2,\ \varphi_1$ and φ_2. Note that the reference sound pressure should be P_{ref}. (10 marks)

(2) As shown in the figure below, there is a tube whose cross-sectional area changes from S_1 to S_2, and only plane waves are traveling in the direction of the x axis through it. Assume that the tubes in ① and ② continue infinitely in the negative and positive directions of the x-axis, respectively, and that there is no effect of reflections from the tube ends.



The instantaneous sound pressure $p_1,\ p_2$ in each region is expressed as follows:

$$① : \ p_1(x,t) = \left(P_i e^{-jkx} + P_r e^{jkx}\right) e^{j\omega t},$$
$$② : \ p_2(x,t) = P_t e^{-jkx} e^{j\omega t},$$

where P_i, P_r, P_t are the amplitudes (real numbers) of the sound waves traveling in the direction of the arrows as shown in the figure. Answer the following questions.

(2-1) Show the volume velocity (product of particle velocity and cross sectional area) $U_1(x,t), U_2(x,t)$ in each region, taking the intrinsic acoustic resistance of the medium as Z_0. (10 marks)

(2-2) Express P_i and P_r using P_t and the area ratio $m = S_2/S_1$, using the condition that the sound pressure and volume velocity are continuous at $x = 0$. (10 marks)

(2-3) Define the sound energy transmission coefficient τ due to cross-sectional area change at $x = 0$ as $\tau = 1 - |P_r/P_i|^2$. In this case, express the attenuation of energy due to the change in cross-sectional area as a positive dB value using m. (10 marks)

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〔Compulsory Question〕

Question III (40 points)

The z-transform of a sequence $x(n)$ is defined as $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$.

The following z-transform pairs can be used. $u(n)$is the unit step sequence.

$x(n)$	$X(z)$	Region of Convergence
$u(n)$	$\dfrac{1}{1-z^{-1}}$, $\dfrac{z}{z-1}$	$\lvert z\rvert > 1$
$-u(-n-1)$	$\dfrac{1}{1-z^{-1}}$, $\dfrac{z}{z-1}$	$\lvert z\rvert < 1$
$a^n u(n)$	$\dfrac{1}{1-az^{-1}}$, $\dfrac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u(-n-1)$	$\dfrac{1}{1-az^{-1}}$, $\dfrac{z}{z-a}$	$\lvert z\rvert < \lvert a\rvert$

(1) $x_1(n)$ and $x_2(n)$ are periodic discrete-time signals with periods N_1 and N_2. Considering that the periodic signals are $x_1(n) = x_1(n + mN_1)$ for any integer m, show the relation between the fundamental period N of the sum signal $x_0(n) = x_1(n) + x_2(n)$ and N_1, N_2.

In addition, prove that the sum of periodic signals of discrete-time signals is always periodic. (15 points)

(2) The discrete-time system T is a linear time-invariant system. When a discrete-time complex signal $x_3(n) = r^n$ is input to this system T, the output is $T\{x_3(n)\} = y_3(n)$. Note that r is a complex number.

When the signal $x_4(n) = x_3(n + n_0)$ is input to the system T, the output is $y_4(n) = r_1 y_3(n)$. Note that n_0 is any integer.

Show r_1, including the derivation process. (5 points)

(3) Prove that the system T given in (2) is $T\{x_3(n)\} = y_3(0)r^n$. (10 points)

(4) Let the discrete-time signal $x_5(n) = (1/2)^n u(n) + (1/3)^n u(n)$. Find the z-transform $X_5(z)$ of this $x_5(n)$, and show the poles, zeros and the region of convergence by the equations. In addition, draw the zero point as ∘, the pole as ×, and the convergence region as a shaded line in the z-plane. (10 points)

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〔Elective Question〕

Question IV (40 points)

Consider the three-degree-of-freedom mechanical vibration system as shown in the figure and answer the following questions. Note and assume that there is no mass of spring, damping due to vibration and motion in the vertical direction are ignored. The mass and spring constant are indicated in the figure.

(1) Suppose the displacements from the equilibrium position of the left mass, middle mass, and the right mass are x_1 (positive to the right), x_2 (positive to the right), and x_3 (positive to the right), respectively, and show the equations of motion on x_1, x_2, and x_3. (8 points)

(2) Suppose that the mechanical vibration system has three normal modes. Since a normal mode represents a vibration of the total system with one frequency, the angular frequency that is common for the three mass displacements is used and let $x_1 = A_1\cos(\omega t+\phi)$, $x_2 = A_2\cos(\omega t+\phi)$, and $x_3 = A_3\cos(\omega t+\phi)$. Let us deduce ω and A_1, A_2, A_3. How can we obtain ω so that $A_1 = A_2 = A_3 = 0$ does not hold? Explain the procedure. (8 points)

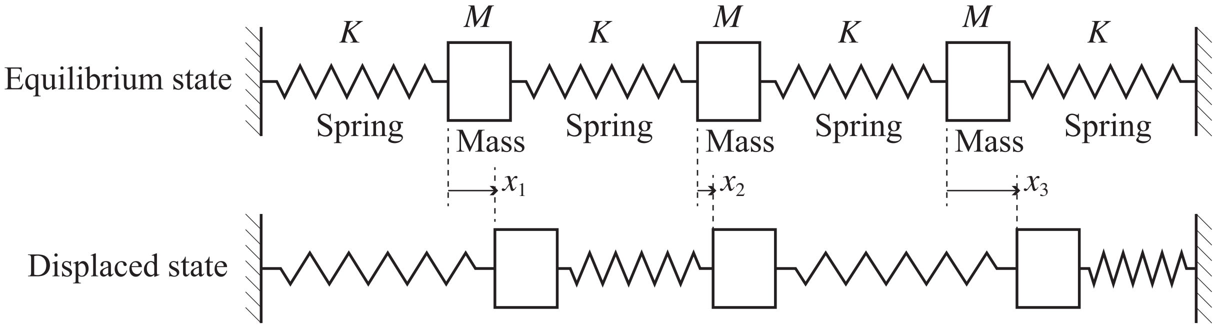
(3) The procedure derived in (2) gives

$$(2K–M\omega^2)^2(2K–M\omega^2)–2(2K–M\omega^2)K^2 = 0.$$

From this, find three solutions with respect to ω, i.e., three normal mode angular frequencies. (8 points)

(4) Assume that the three reference angular frequencies derived in (3) are ω_1, ω_2, and ω_3, in ascending order. What relational expression exists among A_1, A_2, and A_3 when the normal mode angular frequency is ω_1. Explain the procedure for obtaining the relation and show its relational expression. (8 points)

(5) Show the mathematical expressions of the relation among A_1, A_2, A_3 for ω_2 and ω_3, respectively. (8 points)



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〔Elective Question〕

Question V (40 points)

If the energy density in a perfectly diffused room is E [J/m^3], the following equation holds for the energy decay in the room

$$V\frac{dE}{dt} = -\frac{Ec}{4}S\overline{\alpha} \tag{a}$$

where V [m^3] is the volume of the chamber, S [m^2] is the surface area, c [m/s] is the speed of sound, $\overline{\alpha}$ is the average sound absorption coefficient, and t [s] is the time variable.

(1) Define a diffuse sound field. (6 points)

(2) If there are N kinds of materials composing the wall surface of a room, each of which has α_n sound absorption coefficient and S_n [m^2] area ($n = 1,\ 2,\ \cdots, N$), show how the average sound absorption coefficient $\overline{\alpha}$ can be expressed. (6 points)

(3) Explain under what incident conditions the sound absorption coefficient of an acoustical material can be obtained. In addition, regarding the sound absorption coefficient used in equation (a) above, show under what conditions it was measured. (7 points)

(4) The left-hand side of equation (a) above can be interpreted as the rate of change over time of the total energy in the room. Similarly, show the physical meanings of the quantities of $Ec/4$ and $S\overline{\alpha}$ on the right side and the whole equation. (7 points)

(5) The time variation of the energy density obtained by solving equation (a) above can be obtained in the form $E(t) = E_0 e^{-t/\tau}$, where E_0 is the energy density at $t = 0$. Now, express τ using any ones necessary from among $V,\ S,\ \overline{\alpha},\ c,\ E_0$ and derive Sabine’s reverberation formula. (7 points)

(6) In the attenuation process of energy in a room, excess attenuation by air may be considered. Assuming the attenuation coefficient to be β [1/m], explain how Sabine’s reverberation formula is transformed when the sound wave propagates x [m] and the sound intensity decays at a rate of $e^{-\beta x}$. (7 points)

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〔Elective Question〕

Question VI (40 points)

The Fourier transform $X(\omega)$ of a continuous-time signal $x(t)$ is given by the following equation.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

(1) When the Fourier transform of $x(t)$ is $X(\omega)$, show that the Fourier transform of $x(-t)$ is $X(-\omega)$. (8 points)

(2) Let $X(\omega)$ and $Y(\omega)$ are the Fourier transform of $x(t)$ and $y(t)$, respectively, and the convolution of $x(t)$ and $y(t)$ is given by the following equation.

$$z(t) = \int_{-\infty}^{\infty} x(s)y(t-s)ds$$

Show that the Fourier transform of $z(t)$ is $Z(\omega) = X(\omega)Y(\omega)$. (8 points)

(3) Show that the Fourier transform of $f(t) = e^{-\alpha t}u(t)$ is $F(\omega) = \dfrac{1}{\alpha + j\omega}$. Here, $\alpha > 0$ and $u(t)$ is the following step function. (8 points)

$$u(t) = \begin{cases} 1 & (t > 0) \\ 0 & (t < 0) \end{cases}$$

(4) Find the Fourier transform of $g(t) = e^{-\alpha|t|}$ $(\alpha > 0)$ as $G(\omega)$. (8 points)

(5) Find the inverse Fourier transform of $H(\omega) = \dfrac{1}{(1+j\omega)^2}$ as $h(t)$. (8 points)

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〔Elective Question〕

Question VII (40 point)

Answer the following questions. If necessary, use the following values
$\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236, \log_{10} 0.775 = -0.1107$

(1) When using dBV based on an RMS value of 1[V] as an indication of voltage level, explain how to calculate the RMS value [V] of the voltage expressed as -10 dBV, and find the value. (7 points)

(2) As an indication of voltage level, there is dBm, which is based on an RMS voltage of 1 [mW] when applied to a 600 Ω electrical impedance. When using dBm, explain how to calculate the RMS value [V] of a voltage that is 0 dBm, and find the value. (7 points)

(3) As an indication of voltage level, there is dBu, which expresses voltage with reference to the RMS voltage of 0 dBm obtained in (2) above, regardless of the electrical impedance of the load. If dBu is used, explain how to calculate the RMS value [V] of the voltage that is +4 dBu. (7 points)

(4) Suppose there is a device A with a reference level of -10dBV and a device B with a reference level of +4dBu. If the reference level signals are output from each device and input to the same device C at the same gain, what the difference in the levels between the signal from device A and the signal from device B will be? Explain the method of calculating the difference and calculate the value. However, the output impedances of devices A and B shall be sufficiently small compared to the input impedance of device C. In addition, the characteristics of device C are assumed to be linear. (9 points)

(5) In (4), assume that the output of device A is unbalanced and the output of device B is balanced. If the input of device C is balanced, draw a wiring circuit diagram showing what elements and circuits should be used to connect the output terminals of device A to the input terminals of device C. Then, explain the principle of operation of the wiring circuit diagram. (10 points)