Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

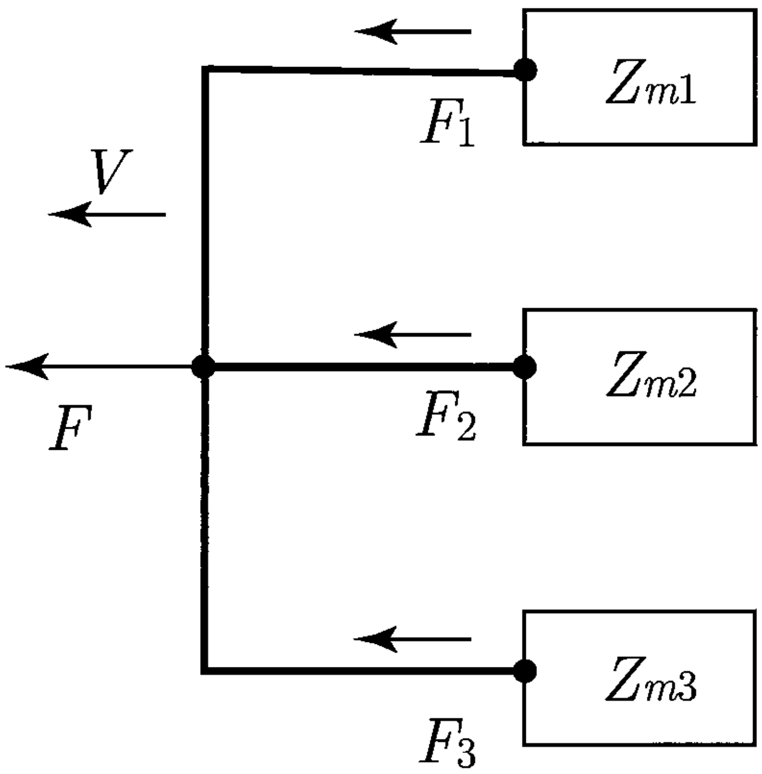
〔Compulsory Question〕

Question I (40 points)

The figure below shows a conceptual diagram of a combination of three mechanical systems.

Let Z_{m1}, Z_{m2}, and Z_{m3} denote the mechanical impedance of each machanical system as viewed from the force point. Each machanical system is combined to move only in the left and right direction with the same velocity V in response to an external force (driving force) F. Where F is $F = F_0 e^{j\omega t}$, ω is the angular frequency of the sinusoidal oscilation [rad/s] , t is the time [s], and j is an imaginary number unit with $j = \sqrt{-1}$.

Answer the following questions.



(1) Let F_1 , F_2 and F_3 be the forces acting on the force points of the respective mechanical systems Z_{m1}, Z_{m2}, and Z_{m3}. Express F_1, F_2 and F_3 using Z_{m1}, Z_{m2}, Z_{m3}, and V. (6 points)

(2) Express the force F that moves the entire combined mechanical system using Z_{m1}, Z_{m2}, Z_{m3} and V.

Also, express the composite mechanical impedance Z_m of the combined mechanical system using Z_{m1}, Z_{m2} , and Z_{m3} . (8 points)

(3) Draw the electrical equivalent circuit of the mechanical system shown in the figure. In doing so, the forces in the mechanical system should correspond to the voltages in the electrical system. In the circuit diagram, draw F, F_1, F_2, F_3, V, Z_{m1}, Z_{m2}, and Z_{m3}. (8 points)

(4) Find the mechanical impedance Z_m of this mechanical system when Z_{m1} is a weight of mass M [kg], Z_{m2} is a spring of compliance C [m/N] and Z_{m3} is a mechanical resistance of resistance R [N · s/m]. (8 points)

(5) Using the solution to the above question, find the frequency f_0 [Hz] of the driving force that minimizes the mechanical impedance using the necessary values of M, C, and R. (10 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕
Question II

Answer the following questions about sound waves. Where j is the imaginary unit of $j = \sqrt{-1}$, ω is the angular frequency, t is the time variable, k is the wavenumber, and x, y are variables representing positions along the orthogonal x, y axes, respectively. Denote the mass density of the medium (air) as ρ_0 and the sound velocity as c, and approximate the value of specific acoustic resistance $Z_0 = \rho_0 c$ to be 400.

(1) Find numerically the amplitude of the sound pressure and the amplitude of the particle velocity for a plane wave vibrating sinusoidallly at 1000 Hz with a sound pressure level of 60 dB. Also indicate the units. (10 points)

(2) In the free field, there exists a pulsating sphere of radius a, where r is the distance from the center of the pulsating sphere, radiating around it a spherical wave whose velocity potential is given by

$$\Phi(r,t) = A\frac{e^{-jkr}}{r}e^{j\omega t},$$

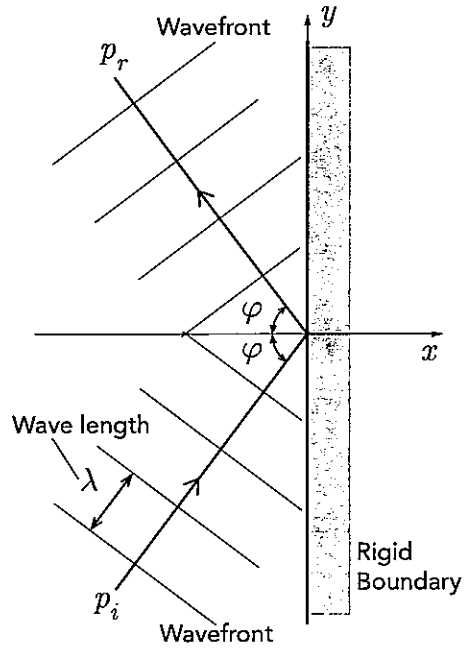
where A is the real amplitude. Find the sound pressure $p(r,t)$ and the particle velocity $u(r,t)$ of the spherical wave generated by this pulsating sphere. In additon, formulate the radiation impedance of the pulsating sphere. (10 points)

(3) Assume a sound field with a rigid boundary at $x = 0$, as shown in the right figure. Here, a plane wave is incident from the direction $x < 0$ at an incidence angle of φ and specularly reflected at a reflection angle φ (wavefront and wavelength λ are shown in the figure). Answer the following questions.

(3-1) The complex sound pressure p_i of the incoming plane wave, with amplitude A, can be expressed as

$$p_i(x,y,t) = Ae^{-j(k_x x + k_y y)}e^{j\omega t},$$

where k_x, k_y are the wave numbers in the x and y directions, respectively. Following this notation, formulate the complex sound pressure $p_r(x,y,t)$ of the reflected wave. (5 points)



(3-2) k_x and k_y are related to the wavenumber k $(= \omega/c = 2\pi/\lambda)$ and $k^2 = k_x^2 + k_y^2$. Express k_x and k_y in terms of k and φ. Briefly show the derivation process as well. (5 points)

(3-3) Show in which direction of the x, y axis the standing wave of sound pressure is observed, and express the distance between the loop and the nodes in the standing wave using the wavelength λ and φ. (5 points)

(3-4) In this sound field, indicate the direction of the sound intensity and show its amplitude using x, y, A, Z_0, and φ. Assume that A is a real number. (5 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question III (40 points)

(III-a)

$x_1(t)$, $x_2(t)$, and $x_3(t)$ are continuous-time signals. Note that t[s] is the time. Also, $x_1(t)$, $x_2(t)$, and $x_3(t)$ are bandlimited signals with the maximum angular frequency Ω_m[rad/s]. The relationship between the discrete-time signal $x(nT_s)$ sampled at time $t = nT_s$ (n: integer) with T_s[s] as the sampling interval in the time domain and its sequence representation $x[n]$ is expressed as follows.

$$x[n] = x(nT_s)$$

Answer the following questions.

(a1) Show the conditions and reasons for the sampling interval T_s to ensure that the number sequence $x_1[n]$ does not lose all the information that the continuous-time signal $x_1(t)$ has. (5 points)

(a2) When the continuous-time signal $x_2(t)$ is a periodic signal with period T_2[s] and T_s satisfies the condition of the above question (a1) for $x_2(t)$, show the relation between period T_2 and T_s required for $x_2[n]$ to be a periodic sequence with period N. (5 points)

(a3) Express a sequence of discrete-time signals $x_3[n]$ when the continuous-time signal $x_3(t)$ is the following. (5 points)

$$x_3(t) = \sin\left(\frac{\pi t}{\sqrt{2}T_s}\right)$$

(a4) Show that $x_3[n]$ in the above question (a3) is a periodic sequence, considering the conditions of the above question (a2). (5 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Compulsory Question〕

Question III (Continued) (40 points)

(III-b)

Answer for the linear time-invariant system represented by the discrete-time difference equation shown in the following equation.

$y[n] = x[n] - 2r(\cos\theta)x[n-1] + r^2 x[n-2]$

(b1) Answer the impulse response of the system. (5 points)

(b2) State the causality of this system. (5 points)

(b3) Answer the transfer function $H(z)$. (5 points)

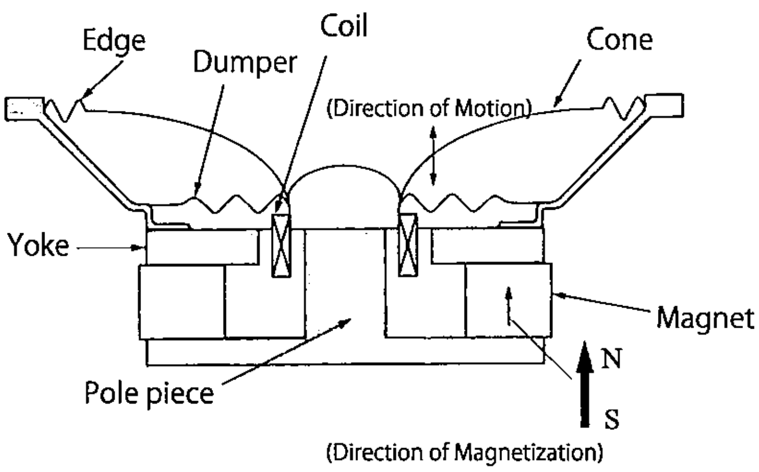
(b4) Answer the poles, zeros, and regions of convergence of the system. (5 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question IV (40 points)

Answer the following questions for the moving coil type electrodynamic transducer shown below.



(1) Assume that the magnetic flux density in the gap between the pole piece and the yoke is B [T] and the length of the coil winding is ℓ [m]. When an alternating current I [A] is applied to this coil, find the electromagnetic driving force F' [N] generated in the vertical direction in the figure. (6 points)

(2) With current I flowing as in the above question, let F [N] be the force externally applied to the vibrating part, z [N·s/m] be the mechanical impedance of the vibrating part, z_0 [N·s/m] be the external impedance as seen from the vibrating part, and V [m/s] be the velocity of the vibrating part.

Express the relationship between F , F', z, z_0 , and V as an equation. (8 points)

(3) Find the voltage E' [V] generated when a coil of winding length ℓ moves at a velocity V [m/s] in a magnetic field of flux density B [T]. In doing so, use the sign $+$ or $-$ to explicitly indicate whether E' is in phase with the current I in (1). (8 points)

(4) With the voltage E' [V] obtained in the above question, if the voltage applied externally to the coil is E [V], the electrical impedance of the electrical circuit containing the coil is Z [Ω], the electrical impedance of an external circuit connected to the coil (for example, the output circuit of a power amplifier) is Z_0[Ω], and the current flowing through the coil is I [A], express the relationship between E, E', Z, Z_0 , and I as an equation. (8 points)

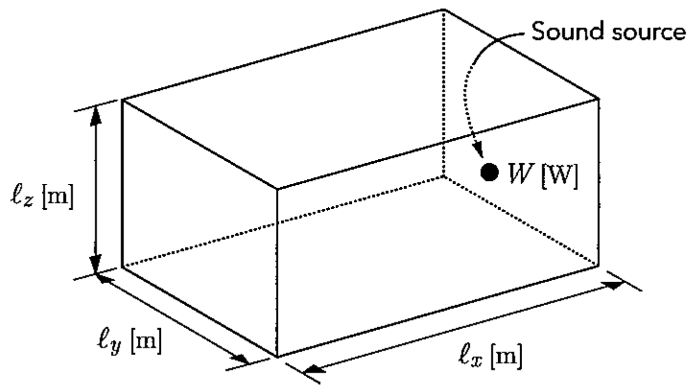
(5) Based on the results of (2) and (4), draw an electrical equivalent circuit representing the relationship between the electrical and mechanical systems of the entire electro-dynamic transducer, using the force coefficient defined by $A \equiv B\ell$ (10 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question V (40 points)

As shown in the figure, there is a small sound source with an acoustic power of W [W] in a rectangular room with dimensions ℓ_x, ℓ_y, ℓ_z [m]. Assuming that the sound field in this room is analyzed geometrically, statistically, and in terms of wave theory, answer the following questions. Let c [m/s] be the speed of sound, and in the answer, the surface area $2(\ell_x \times \ell_y + \ell_y \times \ell_z + \ell_z \times \ell_x)$ may be written as S [m^2] and the volume $\ell_x \times \ell_y \times\ ell_z$ as V [m^3].



(1) Firstly, the sound field is considered geometrically. In particular, if we apply the concept of the mirror image method, there will be one mirror image sound source in one image space because the shape of the room is rectangular. Show an equation that gives the approximate number of mirror image sound sources that occur during the first second after an impulsive source signal is emitted from the sound source. (5 points)

(2) Along with the above mirror image method, the ray-tracing method is a representative method for geometrically predicting the reverberation in this room. Briefly explain the principles and points to note about the method of predicting reverberation by the ray-tracing method. (5 points)

(3) Then, assume that this room is a diffuse field, for simplicity, we consider the statistical prediction of the reverberation of the sound field. Derive an equation for the energy balance that is established in the room under steady state conditions, taking the energy density of the room as E [J/m^3] and the average sound absorption coefficient as $\overline{\alpha}$. (5 points)

(4) In the same situation, assuming a diffuse sound field, the reverberation time was T_1 [s] when an opening of S_1 [m^2] was made in the wall of the room. Find the reverberation time without the aperture using Sabine's reverberation formula and express it using T_1, S_1. (5 points)

(5) Assuming again the situation with no aperture, consider the steady-state sound field in terms of wave theory. In this case, assuming that the boundary surfaces of the room are rigid, the velocity potential in the room sound field is given by the following form

$$\Phi(x, y, z, t) = A \cos k_x x \cdot \cos k_y y \cdot \cos k_z z \cdot e^{j\omega t}.$$

Show which part of this equation represents the eigenmode function and express the equation for the eigenfrequencies (natural frequencies) f_n [Hz] using ℓ_x, ℓ_y, ℓ_z. (10 points)

(6) State what you know about how to count the eigenfrequencies that exist below an arbitrary frequency f [Hz]. (10 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

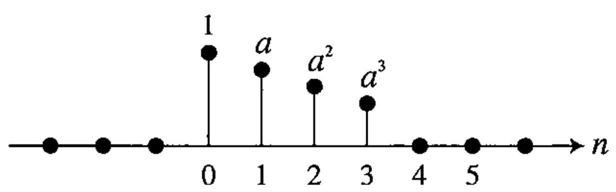
〔Elective Question〕

Question VI (40 points)

Answer the following questions about a digital system which is causal and stable. Note that the system function is given by the following equation when the unit sample response is $h(n)$.

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

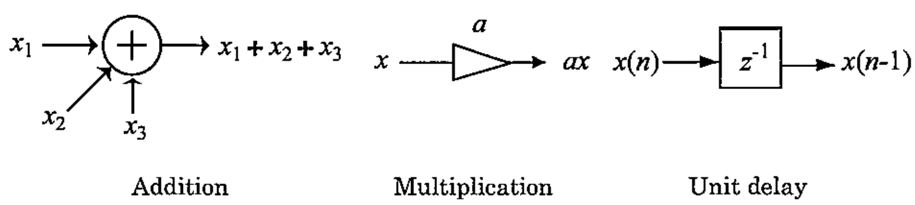
(1) Show the unit sample response $h(n)$ of a system drawn below using the unit sample sequence $\delta(n)$. Here, a is a constant and $0 < a < 1$. (5 points)



(2) Show the system function $H(z)$ for the above unit sample response. In addition, plot the poles and zeros of the system on the z-plane. (10 points)

(3) Show the difference equation of the system. (5 points)

(4) Show the block diagram representation of the system using operators for the addition, multiplication, and unit delay shown below. (5 points)



(5) Suppose that $\hat{H}(z) = 1/H(z)$ is the inverse system of $H(z)$ and $\hat{H}(z)$ is causal. Show the difference equation of the inverse system. (5 points)

(6) Show the unit sample response $\hat{h}(n)$ of the inverse system for $0 \leq n \leq 10$. (5 points)

(7) Explain the stability of the inverse system $\hat{H}(z)$. (5 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

〔Elective Question〕

Question VII (40 points)

The Fourier transform $X(\omega)$ of a continuous-time signal $x(t)$ and the inverse Fourier transform of $X(\omega)$ are given by the following equations ($j = \sqrt{-1}$).

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad ; \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

In addition, the following relations are known for the delta function $\delta(t)$.

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases} \quad ; \quad \int_{-\infty}^{\infty} \delta(t)dt = 1 \quad ; \quad \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt = 1$$

(1) Show that the following relation holds true. (8 points)

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}d\omega$$

(2) $H(\omega)$ is the frequency response of a continuous-time system. Here, K and t_0 are positive constants.

$$H(\omega) = Ke^{-j\omega t_0}$$

When the input signal to the system is $x(t)$, show the output signal $y(t)$ using $x(t)$, K, and t_0. (8 points)

(3) Find the impulse response $h(t)$ of the system from the inverse Fourier transform of frequency response $H(\omega)$. (8 points)

Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

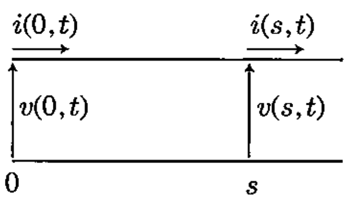
〔Elective Question〕

Question VII (Continued) (40 points)

(4) The figure below shows a distributed parameter line, where i and v represent the current and voltage, respectively. Under the steady state, the propagation constant is given by the following equation.

$$\gamma(\omega) = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Here, R, L, G, and C are the series resistance, series inductance, parallel conductance, and parallel capacitance, respectively, per unit length. If the relation $L/R = C/G$ holds, the propagation constant can be expressed as $\gamma(\omega) = \alpha + j\omega\beta$, where α and β are real number parameters.



(4-1) Express each of the real number parameters α and β using R, L, G, and C. (8 points)

(4-2) Suppose that the voltage at the origin of the distributed parameter line is the input and the voltage at a point, at which the distance from the origin is s, is the output. Then the frequency response between these two points is $H(\omega) = e^{-\gamma(\omega)s}$. When the relation $L/R = C/G$ holds, this frequency response can be expressed as $H(\omega) = Ke^{-j\omega t_0}$ in agreement with the question (2) above. In this case, show K and t_0 using R, L, G, C, and s. (8 points)