Acoustic Engineering / Signal Processing

Example Answers

Examinee's number

Use a separate answer sheet for each question.

Question I

(1)

$$D = \frac{F/M}{\omega_0^2 - \omega^2 + jh\omega}$$

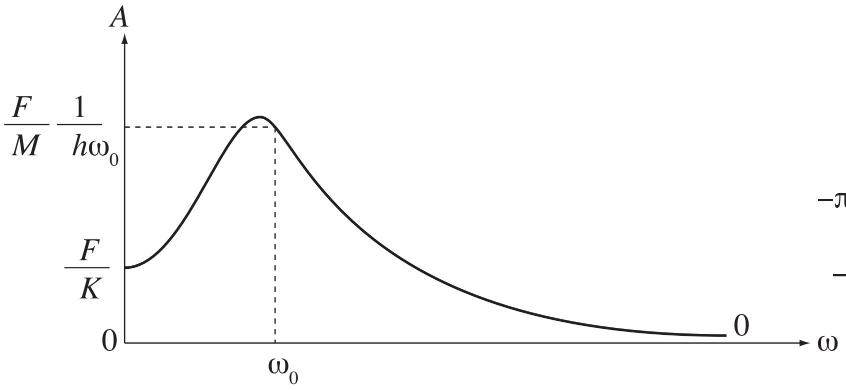
(2)

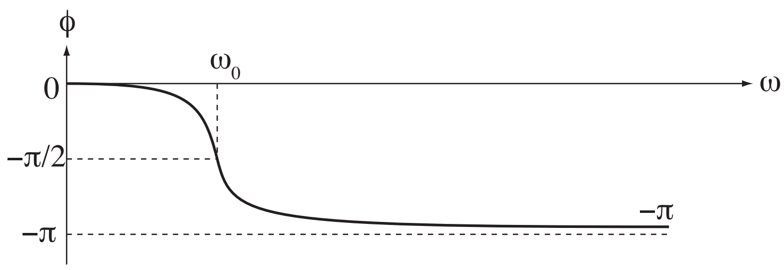
$$A = \frac{F/M}{\sqrt{(\omega_0^2 - \omega^2)^2 + (h\omega)^2}}$$

$$\cos\phi = \frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (h\omega)^2}}$$

$$\sin\phi = \frac{-h\omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + (h\omega)^2}}$$

(3)





(4)

$$\omega_{max} = \sqrt{\omega_0^2 - h^2/2}$$

$$A_{max} = \frac{F/M}{h\sqrt{\omega_0^2 - h^2/4}}$$

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Question I (Continued)

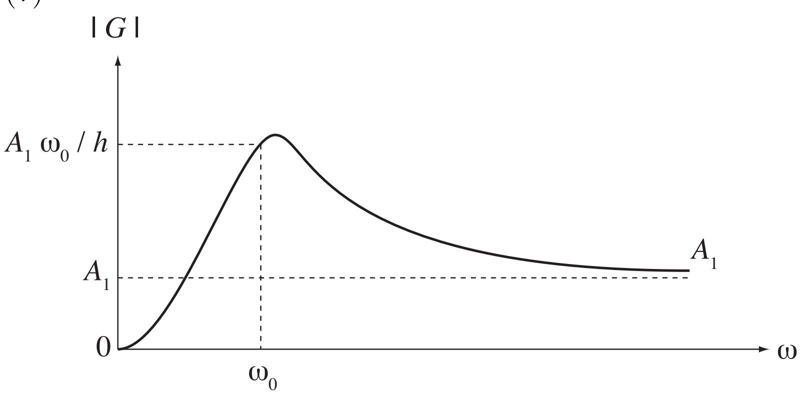
(5)

$$D_2 = \frac{\omega_0^2 + jh\omega}{\omega_0^2 - \omega^2 + jh\omega} A_1$$

(6)

$$G = \frac{\omega^2}{\omega_0^2 - \omega^2 + jh\omega} A_1$$

(7)



Acoustic Engineering / Signal Processing	Example Answers (page 3 of 9)	Examinee's number

Use a separate answer sheet for each question.

Question II

(1) Given that $u = -\dfrac{\partial \phi}{\partial x}$, the dimensional analysis gives $[\mathrm{m/s}] = \dfrac{\phi}{[\mathrm{m}]}$. Therefore, the unit of ϕ is $\mathrm{m^2/s}$.

(2) The sound pressure is given by $p(x,t) = \rho_0 \dfrac{\partial \phi}{\partial t} = -\rho_0 A\omega \sin(\omega t - kx)$, and the particle velocity is $u(x,t) = -\dfrac{\partial \phi}{\partial x} = -Ak\sin(\omega t - kx)$.

(3) The instantaneous sound intensity $I(x,t)$ is $I(x,t) = p(x,t)u(x,t) = \rho_0 \omega k A^2 \sin^2(\omega t - kx)$. The sound intensity is obtained by integrating this over one period.

$$I(x) = \frac{1}{T}\int_0^T \rho_0 \omega k A^2 \sin^2(\omega t - kx)dt = \frac{\rho_0 \omega k A^2}{2T}\int_0^T [1 - \cos 2(\omega t - kx)]\,dt = \cdots = \frac{\rho_0 \omega k A^2}{2}$$

(4) The sound pressure level (SPL) is given by $10\log_{10}(\overline{P}/P_{\mathrm{ref}})^2$, and the sound intensity level (IL) is given by $10\log_{10}(I/I_{\mathrm{ref}})$, where $\overline{P}$ is the root mean square of $p(t)$. For a sinusoidal wave, $\overline{P} = |p(t)|/\sqrt{2}$, P_{ref} is the reference sound pressure: 2×10^{-5} Pa, and I_{ref} is the reference sound intensity: 10^{-12} $\mathrm{W/m^2}$.

(4-1) $\therefore \mathrm{SPL} = 10\log_{10}\left(\dfrac{\rho_0^2 A^2 \omega^2/2}{P_{\mathrm{ref}}^2}\right)$, $\mathrm{IL} = 10\log_{10}\left(\dfrac{\rho_0 \omega k A^2/2}{I_{\mathrm{ref}}}\right)$

(4-2) If the two levels are equal, then $\dfrac{\rho_0 c}{P_{\mathrm{ref}}^2} = \dfrac{1}{I_{\mathrm{ref}}}$ holds.

$$\therefore P_{\mathrm{ref}}^2 = 4 \times 10^{-10} = 400 \times 10^{-12} = \rho_0 c \times I_{\mathrm{ref}} = \rho_0 c \times 10^{-12}$$

This approximates that $\rho_0 c = 400$. Under typical room temperature conditions (around 15°C), this value is approximately 415. Since $c \approx 331.5 + 0.61 \times \text{Temperature}$, the higher the temperature, the less accurate this approximation becomes. In other words, the approximation holds better at lower temperatures.

(5) When two waves are superimposed, the result is:

$$\phi(x,t) + \phi'(x,t) = \mathrm{Re}\left[(Ae^{-jkx} + Be^{jkx})e^{j\omega t}\right] = \cdots = \mathrm{Re}\left[\sqrt{A^2 + B^2 + 2AB\cos 2kx} \cdot e^{j\theta} e^{j\omega t}\right]$$

Thus, the maximum amplitude becomes $|A + B|$. Therefore, the amplitude of the combined sound pressure is $\rho_0(A+B)\omega$. The difference in sound pressure level is given by:

$$10\log_{10}\left(\frac{\rho_0^2 (A+B)^2 \omega^2/2}{P_{\mathrm{ref}}^2}\right) - 10\log_{10}\left(\frac{\rho_0^2 A^2 \omega^2/2}{P_{\mathrm{ref}}^2}\right) = 10\log_{10}\left[\frac{(A+B)^2}{A^2}\right]$$

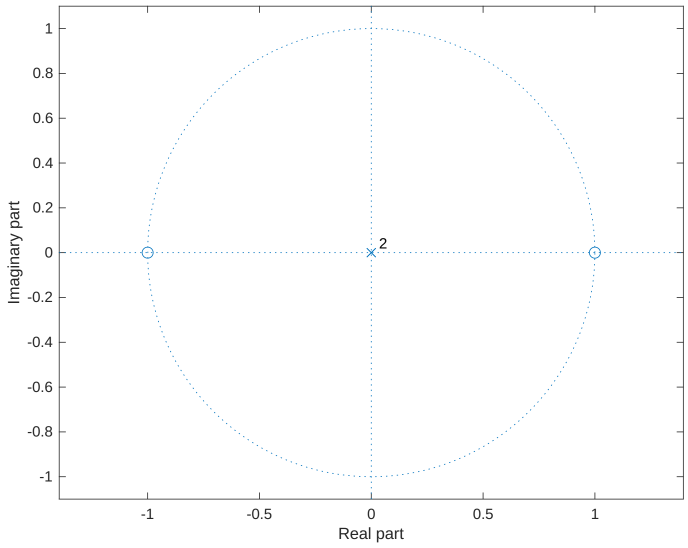
Therefore, the sound pressure level increases by a maximum of $10\log_{10}\left[\dfrac{(A+B)^2}{A^2}\right]$ [dB].

Use a separate answer sheet for each question.

Question III

(1) $H(z) = 1 - z^{-1}$

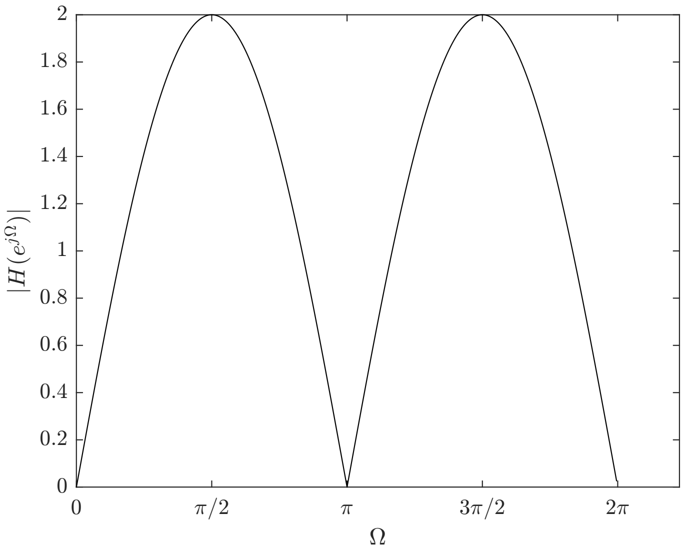
(2) Pole points $z = 0$ Zero points $z = \pm 1$

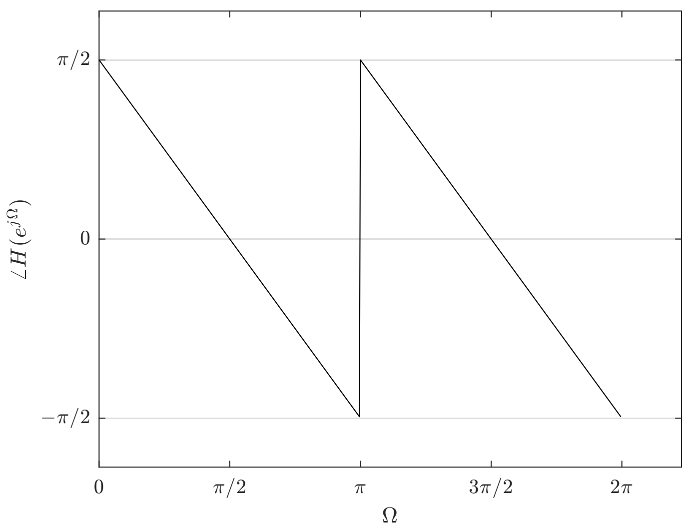


(3) $H(e^{j\Omega}) = 2e^{-j(\Omega-\pi/2)} \sin \Omega$

$|H(e^{j\Omega})| = |2 \sin \Omega|$

$\angle H(e^{j\Omega}) = -\Omega + \pi/2$





Question III (Continued)

(4) Low-pass frequency = 4 kHz

(5) $0 \geq f < 4$ kHz

The amplitude frequency response $2|\sin(\pi \times f \times 10^{-3}/4)|$

The phase frequency response $-(\pi \times f \times 10^{3}/4) + \pi/2$

$f \geq 4$ kHz

The amplitude frequency response 0

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Use a separate answer sheet for each question.

Question IV

(1) $F' = IB\ell$

(2) $F' = 1.92$ N

(3) $V = 0.153$ m/s

(4) $X = 0.122$ mm

(5) $Z_{em} = -j \cdot 0.587\ \Omega$

(6) $Z_{em} = 0.2 - j \cdot 0.6\ \Omega$

Acoustic Engineering / Signal Processing	Example Answers (page 7 of 9)	Examinee's number

Use a separate answer sheet for each question.

Question V

(1) **Diffuse sound field:** A sound field in which the energy density is the same at all points and the sound energy arrives from all directions with equal probability at any given point.
Reverberation time: The time it takes for the energy in the sound field to decay to one-millionth of its original value after the supply of energy into the sound field has ceased. In terms of level, it is the time required for a 60 dB decay.

(2) $V\dfrac{dE(t)}{dt} = -V\dfrac{E(t)}{\tau}$, $\therefore \dfrac{dE}{E} = -\dfrac{dt}{\tau}$, $\therefore \ln E = -\dfrac{t}{\tau} + C$ (constant). Assuming $E(0) = E_0$ at $t = 0$, we get:
$\ln E_0 = C$, $\therefore E(t) = E_0 e^{-t/\tau}$. Letting T be the reverberation time, we define it by $E(T) = E_0 e^{-T/\tau} = E_0 \cdot 10^{-6}$.
Thus, $T = \tau 6 \ln 10$.

(3) **Mean free path:** The average distance between the next impact of a particle or sound ray with sound energy emitted from a sound source after it has impacted a room boundary.
In this room, the following relation holds: $-\dfrac{VE}{\tau} = -\dfrac{EcS\overline{\alpha}}{4}$, and hence, $\dfrac{1}{\tau} = \dfrac{cS\overline{\alpha}}{4V}$. From this, we get $\dfrac{c\overline{\alpha}}{\ell_m} = \dfrac{cS\overline{\alpha}}{4}$, thus $\ell_m = \dfrac{4V}{S}$.

(4) The power balance equation for a sound source with acoustic power W [W] is given by: $W - V\dfrac{E(t)}{\tau} = V\dfrac{dE(t)}{dt}$.
In the steady state, the right-hand side becomes zero. Using $\dfrac{1}{\tau} = \dfrac{cS\overline{\alpha}}{4V}$, we get $W = V\dfrac{cS\overline{\alpha}}{4V}E$. Dividing both sides by the reference value $W_0 = E_0 c A_0$, we have:

$$\frac{W}{W_0} = \frac{EcS\overline{\alpha}}{4W_0} = \frac{ES\overline{\alpha}}{E_0 A_0}\cdot\frac{1}{4}, \quad \therefore 10\log_{10}\left(\frac{W}{W_0}\right) = 10\log_{10}\left(\frac{E}{E_0}\right) + 10\log_{10}\left(\frac{S\overline{\alpha}}{A_0}\right) - 10\log_{10}4$$

$$\therefore \mathrm{PWL} = \mathrm{SPL} + 10\log_{10}\left(\frac{S\overline{\alpha}}{A_0}\right) - 6$$

(5) The room constant R is given by: $R = \dfrac{S\overline{\alpha}}{1-\overline{\alpha}}$. The first term inside the parentheses on the right-hand side represents the direct component arriving at the receiver from the source, while the second term represents the energy of the reverberant (or diffuse) component formed based on the 'original' energy, which is the energy emitted from the sound source until it first encounters a boundary.

(6) $T_0 = \dfrac{KV}{S\overline{\alpha}}$, $T_1 = \dfrac{KV}{(S-S_1)\overline{\alpha} + 0.5S_1} = \dfrac{KV}{S\overline{\alpha} + S_1(0.5-\overline{\alpha})}$. Solving this system yields: $S_1 = \dfrac{T_0 - T_1}{T_1}\cdot\dfrac{2S\overline{\alpha}}{1-2\overline{\alpha}}$.

Acoustic Engineering / Signal Processing

Example Answers

Examinee's number

Use a separate answer sheet for each question.

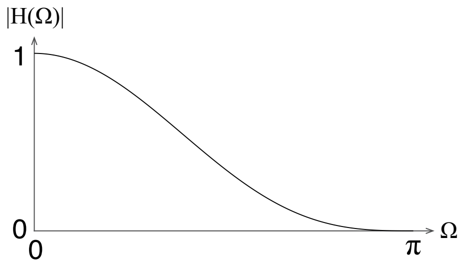
Question VI

(1) $h(n) = \frac{1}{8}\{\delta(n) + 3\delta(n-1) + 3\delta(n-2) + \delta(n-3)\}$

(2) $H(\Omega) = \frac{1}{8}\left\{1 + 3e^{-j\Omega} + 3e^{-2j\Omega} + e^{-3j\Omega}\right\}$

(3) The frequency response can be written as $H(\Omega) = \cos^3\left(\frac{1}{2}\Omega\right) e^{-\frac{3}{2}j\Omega}$. Then the amplitude and phase responses are $|H(\Omega)| = \cos^3\left(\frac{1}{2}\Omega\right)$ and $\angle H(\Omega) = -\frac{3}{2}\Omega$, respectively.

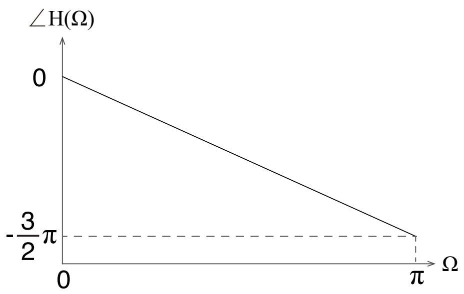
(4)



$|H(0)| = 1$, $|H(\pi)| = 0$

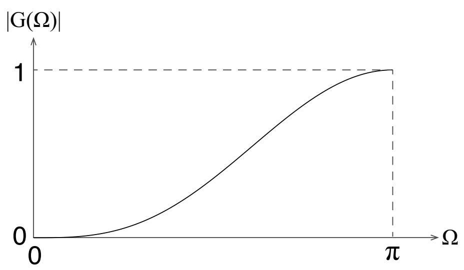
The figure shows that this system is a low-pass filter which passes low frequency components.

(5)



(6) $X_2(\Omega) = \sum_{n=0}^{N-1} x_2(n)e^{-j\Omega n} = \sum_{n=0}^{N-1}(-1)^n x_1(n)e^{-j\Omega n} = \sum_{n=0}^{N-1} x_1(n)e^{j\pi n}e^{-j\Omega n} = \sum_{n=0}^{N-1} x_1(n)e^{-j(\Omega-\pi)n} = X_1(\Omega - \pi)$

(7)



The figure shows that this system is a high-pass filter which passes high frequency components.

Use a separate answer sheet for each question.

Question VII

(1) $X^{'}(\omega) = \int_{-\infty}^{\infty} x^{'}(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} x(t-t_0)e^{-j\omega t}dt = \int_{-\infty}^{\infty} x(t)e^{-j\omega(t+t_0)}dt = e^{-j\omega t_0}\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = e^{-j\omega t_0}X(\omega)$

(2) It is shown below that the Fourier transform of $e^{j\omega_0 t}x(t)$ is $X(\omega-\omega_0)$.
$\int_{-\infty}^{\infty} e^{j\omega_0 t}x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} x(t)e^{-j(\omega-\omega_0)t}dt = X(\omega-\omega_0)$

(3) When $x(t)=1$ in question (2), the Fourier transform of $e^{j\omega_0 t}$ is $2\pi\delta(\omega-\omega_0)$ and the Fourier transform of $e^{-j\omega_0 t}$ is $2\pi\delta(\omega+\omega_0)$. Furthermore, from the relationship $f(t) = \frac{1}{2}(e^{j\omega_0 t}+e^{-j\omega_0 t})$, we obtain the Fourier transform $F(\omega) = \pi\left\{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right\}$.

(4) From the relationship $\cos^2(\omega_0 t) = \left\{\frac{1}{2}(e^{j\omega_0 t}+e^{-j\omega_0 t})\right\}^2 = \frac{1}{4}\left\{e^{2j\omega_0 t}+2+e^{-2j\omega_0 t}\right\}$, we obtain the Fourier transform $G(\omega) = \frac{1}{2}\pi\left\{\delta(\omega-2\omega_0)+2\delta(\omega)+\delta(\omega+2\omega_0)\right\}$.

(5) $S(\omega) = \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \frac{1}{T_0}e^{-j\omega t}dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{-j\omega t}dt = -\frac{1}{jT_0\omega}\left\{e^{-j\frac{T_0}{2}\omega}-e^{j\frac{T_0}{2}\omega}\right\} = \frac{2}{T_0\omega}\sin(\frac{T_0}{2}\omega)$

(6) $S(\omega)$ is the sinc function and $S(0)=1$. $S(\omega)=0$ when $\frac{T_0}{2}\omega = \pm\pi, \pm 2\pi, \pm 3\pi, ...$, and $\frac{T_0}{2}\omega_1 = \pi$ and $\frac{T_0}{2}\omega_2 = -\pi$. Therefore, $\Delta\omega = \omega_1-\omega_2 = \frac{4\pi}{T_0}$ is inversely proportional to T_0.

Acoustic Engineering / Signal Processing

Intention of the Exam Questions

Question I

To assess whether examinees understand how to analyze governing equations of single-degree-of-freedom mass-spring-damper systems.

Question II

This question aims to assess whether examinees understand the relationships between velocity potential, sound pressure, particle velocity, and sound intensity based on the wave equation, as well as whether they can perform basic calculations involving decibels (dB).

Question III

This question measures the level of understanding of digital signal processing among examinees, as well as their understanding of the relationship between discrete-time signal processing and continuous-time signal processing.

Question IV

To assess whether the examinees understand the mechanism of a moving coil-type electrodynamic transducer.

Question V

This question assesses the understanding of the properties of a diffuse sound field, a fundamental concept in architectural acoustics, and whether the reverberation time—a representative acoustic physical index—can be calculated using Sabine's reverberation formula.

Question VI

This question aims to assess the level of understanding of digital signal processing required for the master's degree research.

Question VII

This question aims to assess the level of understanding of acoustic signal processing required for the master's degree research.