

Acoustic Engineering / Signal Processing

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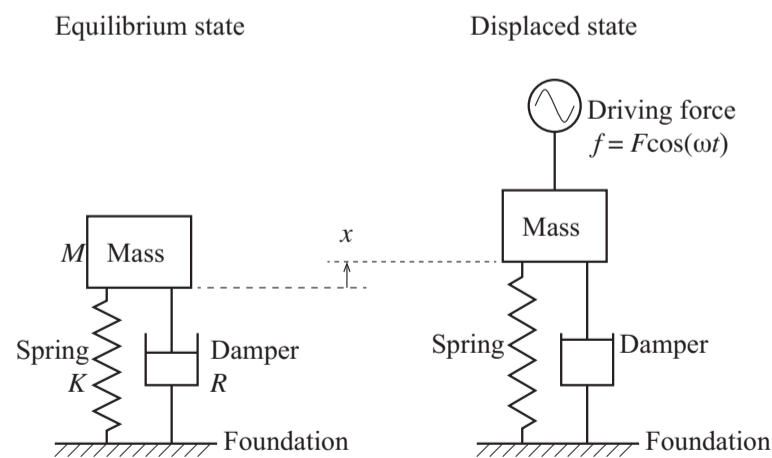
Notes: Answering questions I, II, and III is compulsory. In addition, you must select and answer two questions from among the elective questions IV, V, VI and VII. Do not write on the back side of the answer sheet, or your answers will not be marked. Use a separate answer sheet for each question.

{Compulsory Question}

Question I (40 points)

The figure below shows a mechanical vibration system driven by an external force. Note that x is the displacement of the mass, M is the mass, R is the resistance of the damper, K is the stiffness of the spring, F is the driving force amplitude, ω is the angular frequency, and t represents the time. One end of the spring and that of the damper are fixed to a perfectly rigid foundation. Assume that its damping is light, there is no mass of spring, and no mass of damper. The symbol j denotes an imaginary unit and can be used in the answers. Answer the following questions.

- (1) An important characterization of mechanical vibration systems is the mechanical impedance. Briefly explain what kind of physical quantity the mechanical impedance is and what characteristics of mechanical vibration systems it represents. (5 points)
- (2) Write down the expression of the mechanical impedance of the system below, using necessary ones from t , M , R , K , and ω . (5 points)



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[Compulsory Question]

Question I (Continued) (40 points)

Suppose that the steady-state solution of $x(t)$ is expressed as

$$x(t) = A \cos(\omega t + \phi),$$

where

$$A = \frac{F}{M} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}},$$

and ω_0 is the natural angular frequency of the mechanical vibration system, $\gamma (= R/M)$ is the damping coefficient, and ϕ is the initial phase.

In addition, when the amplitude of the force transmitted to the foundation from the mechanical vibration system is given by F_B , the transmissibility τ of the mechanical vibration system is defined by $\tau = F_B/F$.

- (3) Write down the force $f_S(t)$ transmitted to the foundation from the spring and the force $f_D(t)$ transmitted to the foundation from the damper, using necessary ones from t , A , M , R , K , ω , and ϕ . (6 points)
- (4) The value F_B is the amplitude of a single sinusoidal wave equivalent to $f_S(t) + f_D(t)$. Write down the expression of F_B , using necessary ones from t , A , M , R , K , ω , and ϕ . (6 points)
- (5) The value τ can be represented using only ω , ω_0 , and γ . Write down the expression. (6 points)
- (6) Suppose that a value of ω is given. In order to keep τ below 1 for the given ω , how can we adjust M , R , and K ? Explain the process using mathematical expressions. (6 points)
- (7) Draw a rough outline of a frequency characteristic graph of τ (with ω on the horizontal axis and τ on the vertical axis). Explicitly give the values of τ at $\omega = 0$, $\omega = \omega_0$, and $\omega \rightarrow \infty$. (6 points)

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[Compulsory Question]

Question II (40 points)

Answer the following questions about sound waves.

- (1) Calculate the sound pressure level and the intensity level of a plane sine wave with an amplitude of 0.2 Pa. Show the derivation process. You may approximate the characteristic acoustic impedance of the medium as 400. (7 points)
- (2) When the sound pressure at a point on the x axis is $p(x)$ and the particle velocity along the x axis is $u_x(x)$, explain that the equation describing the motion in the x direction caused by the sound wave is given by the following, where ρ_0 is the mass density of the medium. (8 points)

$$\frac{\partial p}{\partial x} + \rho_0 \frac{\partial u_x}{\partial t} = 0$$

- (3) Using the above equation, explain how to estimate particle velocity by measuring sound pressure. Also mention the limits of measurement accuracy. (8 points)
- (4) Show the definition of the vertical incident sound absorption coefficient of acoustic materials and explain its typical measurement method. (7 points)
- (5) In a free sound field, there is a small sound source with a certain power level, and the sound pressure level at a receiving point at a distance r from the sound source is L [dB]. If another sound source with the same power level is placed at a distance of $2r$ from the receiving point and both are operated simultaneously, show how much the level observed at the receiving point increases from L . Assume that the sounds emitted from the two sound sources do not interfere with each other. The answer may be expressed in logarithmic notation. (10 points)

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[Compulsory Question]

Question III (40 points)

(III-a)

Solve the following discrete-time signal processing problems using only the time domain. Do not solve in the z domain. Note that n represents time, and $u[n]$ is the unit step sequence shown below.

$$u[n] = \begin{cases} 0 & (n < 0) \\ 1 & (n \geq 0) \end{cases}$$

The convolution of sequences $x[n]$ and $y[n]$ is given by the following equation.

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

The following formula may also be used. Note that α in the formula is a real number.

$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1 - \alpha^N}{1 - \alpha} & (\alpha \neq 1) \\ N & (\alpha = 1) \end{cases}$	$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1 - \alpha} \quad (\alpha < 1)$
$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1 - \alpha} \quad (\alpha < 1)$	$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1 - \alpha)^2} \quad (\alpha < 1)$

- (1) Consider a discrete-time linear time-invariant system with impulse response $h_1[n] = \alpha_1^{n-1}u[n-1]$. Show whether this system is causal or not, and explain your reasoning. Note that α_1 is an arbitrary real number. (10 points)
- (2) Show that the system in the previous question (1) is Bounded-Input Bounded-Output Stability using α_1 and include the explanation process. (5 points)
- (3) Consider a discrete-time linear time-invariant system with impulse response $h_2[n] = \beta_2^n u[n]$. Calculate the output $y_2[n]$ when the signal $x_2[n] = \alpha_2^n u[n]$ is input to this linear time-invariant system, including the calculation process. Note that α_2 and β_2 are arbitrary real numbers. (10 points)

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{Compulsory Question}

Question III (Continued) (40 points)

- (4) Consider a discrete-time linear time-invariant system whose impulse response is $h_3[n] = \alpha_3^{-n}u[-n]$. Show the output $y_3[n]$ for $n \leq 0$ when the signal $x_3[n] = \alpha_3^n u[n]$ is input to this linear time-invariant system, including the calculation process. Note that α_3 is a real number such that $0 < \alpha_3 < 1$. (Pay attention to $u[-n]$.) (5 points)
- (5) Given the system and input in the previous question (4), show the output $y_3[n]$ for $n > 0$ including the calculation process. (5 points)
- (6) Given the system and input in the previous questions (4) and (5), show the output $y_3[n]$ for all n in a single equation. Do not use case distinctions for n . Also, show whether the system is causal or not, and explain your reasoning. (5 points)

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[Elective Question]

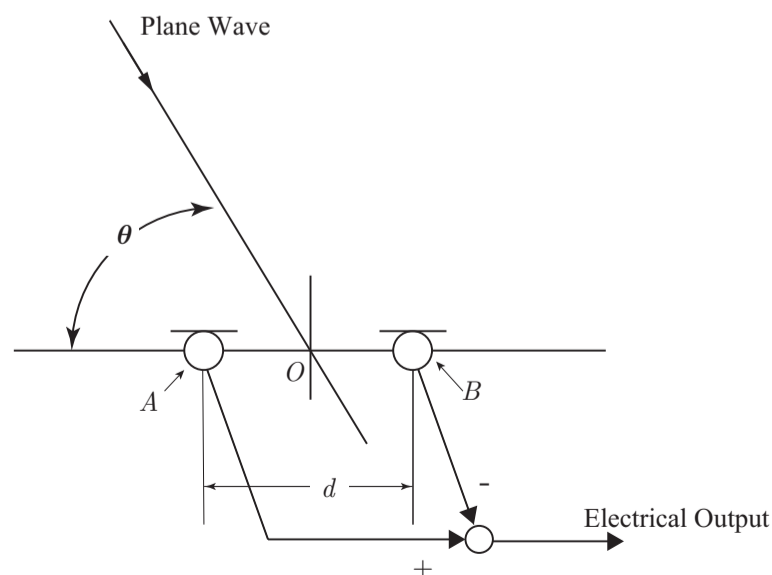
Question IV (40 points)

As shown in the figure below, consider a system that uses two omnidirectional microphone elements A and B placed at a distance d [m] in a free sound field to receive plane waves, and subtract the output voltages to extract the difference.

Let the angle between the line connecting the two microphones and the direction of propagation of the plane wave be θ [rad]. and the midpoint of the two microphone elements be O . Let the sensitivity of each microphone be K [V/Pa].

The incident plane wave is a pure tone, with the effective sound pressure P [Pa], angular frequency ω [rad/s], and sound speed c [m/s]. The wavenumber is $k = \omega/c$. The instantaneous sound pressure at point O is expressed as $\sqrt{2}Pe^{j\omega t}$. Here, j is the imaginary unit.

- (1) When a microphone with sensitivity K is placed at the origin O to receive sound, express the instantaneous value of the output voltage $\phi_0(t)$. (5 points)
- (2) With respect to a plane wave whose wavefront passes through the origin O at a certain time, what is the time delay, expressed as a positive value, at which the wavefront passes through microphones A and B ? (5 points)
- (3) Express the instantaneous value $\phi(t)$ of the signal obtained by subtracting the output voltage of microphone B from that of microphone A . (10 points)
- (4) Find the absolute value of the ratio of $\phi(t)$ to $\phi_0(t)$. Normalize the maximum value of D to 1. (10 points)
- (5) Draw a graph of D as a function of θ when $kd = \pi$. (10 points)



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{Elective Question}

Question V (40 points)

Consider a design of the reverberation time T based on the diffuse sound field for a rectangular room with floor dimensions $l_x = 8$ m, $l_y = 9$ m, and ceiling height $l_z = 2.88$ m. Answer the following questions. Note that the total surface area of the room is $S = 2(l_x l_y + l_y l_z + l_z l_x)$ [m²], and the total volume is $V = l_x l_y l_z$ [m³]. Define the average sound absorption coefficient as the weighted arithmetic mean of the areas of each surface in the room, and denote its value as $\bar{\alpha}$. When calculating the actual values, use the sound velocity c [m/s] and calculate using the equation $K := 24 \ln 10 / c = 0.161$ s/m.

- (1) Define diffuse sound fields and reverberation time. (7 points)
- (2) Derive Sabine's reverberation formula. It is permissible to use the fact that the normal sound intensity incident on the interior surface is $\frac{Ec}{4}$ [J/(m²s)] in a diffuse sound field, where E [J/m³] denotes the energy density inside the room. (6 points)
- (3) Derive Eyring's reverberation formula. In a diffuse sound field, it is permissible to use the mean free path expressed as $\frac{4V}{S}$ [m]. (6 points)

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{Elective Question}

Question V (Continued) (40 points)

(4) Consider the average sound absorption coefficient required to achieve a reverberation time $T = 0.4$ s. Perform intermediate calculations by converting decimals to equivalent irreducible fractions, and give your answers as irreducible fractions.

(4-1) Calculate the value of $\frac{V}{S}$ numerically. (1 point)

(4-2) Calculate the average sound absorption coefficient numerically based on Sabine's reverberation formula. (2 points)

(4-3) Calculate the average sound absorption coefficient numerically based on Eyring's reverberation formula and the following approximation. (4 points)

$$-\ln(1 - x) \approx x + \frac{1}{2}x^2 \quad (\text{a})$$

(4-4) Explain the expected results when designing based on Sabine's reverberation formula for these situations. (4 points)

(5) Adjust the reverberation using material P with a statistical sound absorption coefficient of $\alpha = 0.84$. At this time, verify whether a reverberation time of $T = 0.46$ s can be achieved by designing the ceiling surface as the only sound-absorbing surface using material P and the other five surfaces as reflective surfaces with a statistical sound absorption coefficient of 0.

(5-1) Verify based on Sabine's reverberation formula. (4 points)

(5-2) Verify based on Eyring's reverberation formula and the approximation in the above (a). (6 points)

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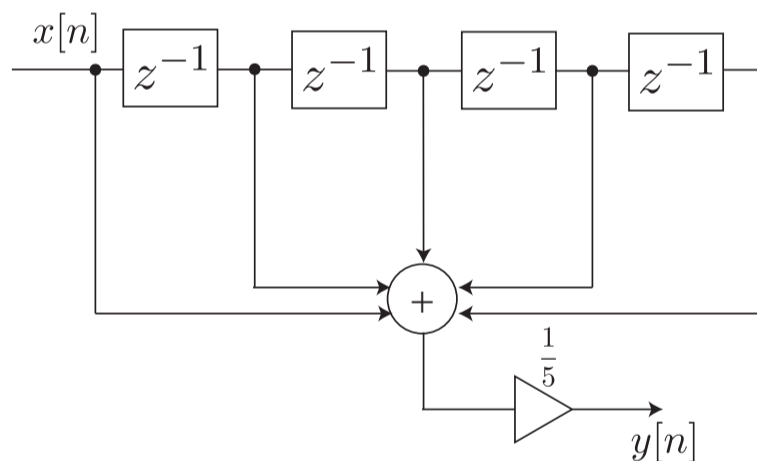
{Elective Question}

Question VI (40 points)

The figure below shows a block diagram of a discrete-time linear time-invariant system with causality. Here, $x[n]$ is the input signal to the system, $y[n]$ is the signal produced by the system as its output, and z^{-1} represents a unit delay. The Fourier transform $X(\Omega)$ of the discrete-time signal $x[n]$ is given as follows, where j denotes the imaginary unit.

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

Please answer the following questions.



- (1) Determine the impulse response $h[n]$ of the system. (6 points)
- (2) Express $y[n]$ and $y[n - n_0]$ in terms of $x[n]$. (6 points)
- (3) Determine the frequency response $H(\Omega)$ of the system. (6 points)

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{Elective Question}

Question VI (Continued) (40 points)

- (4) From the frequency response $H(\Omega)$, find the amplitude response $|H(\Omega)|$ and the phase response $\angle H(\Omega)$ for $0 \leq \Omega \leq \pi$. Note that the value of $\angle H(\Omega)$ must satisfy the condition $-\pi \leq \angle H(\Omega) \leq \pi$. You may also use the following identities: $\cos \frac{\pi}{5} = \frac{1 + \sqrt{5}}{4}$, $\cos \frac{2\pi}{5} = \frac{-1 + \sqrt{5}}{4}$. (6 points)
- (5) Sketch the amplitude response $|H(\Omega)|$ over the range $0 \leq \Omega \leq \pi$. Be sure to indicate the values of $|H(0)|$, $|H(\pi)|$, and the frequency Ω at which $|H(\Omega)| = 0$ in your diagram. (5 points)
- (6) Plot the phase response $\angle H(\Omega)$ for $0 \leq \Omega \leq \pi$. (5 points)
- (7) By expressing $y[n]$ in terms of $y[n - 1]$, a system with the same characteristics as the original can be realized in pole-zero form. Draw the block diagram of the system in this pole-zero representation. (6 points)

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[Elective Question]

Question VII (40 points)

The following equation represents a continuous-time causal linear time-invariant system.

$$y(t) = x(t) + gx(t - T) \quad \dots\dots (a)$$

Here, $x(t)$ and $y(t)$ are the input and output of the system, g is a real number, and T is a positive integer. Note that the Fourier transform $X(\omega)$ of a continuous-time signal $x(t)$ and the inverse Fourier transform of $X(\omega)$ are given by the following equations.

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \quad ; \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$

Here, j is the imaginary unit. In addition, the following relations are known for the delta function $\delta(t)$.

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases} \quad ; \quad \int_{-\infty}^{\infty} \delta(t) dt = 1 \quad ; \quad \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t} dt = 1$$

The last equation shows that the Fourier transform of $\delta(t)$ is 1.

- (1) Let $X(\omega)$ be the Fourier transform of a signal $x(t)$. Find the Fourier transform of $x(t - s)$ as $X'(\omega)$ using $X(\omega)$. Here, s is a real number. (8 points)
- (2) For the system of equation (a), let $X(\omega)$ be the Fourier transform of $x(t)$ and $Y(\omega)$ be the Fourier transform of $y(t)$. Find the frequency response of the system as $H(\omega)$. (8 points)
- (3) The auto-correlation function $\phi(\tau)$ of a signal $x(t)$ is defined as $\phi(\tau) = \int_{-\infty}^{\infty} x(t)x(t - \tau)dt$. In addition, $\phi(\tau)$ is equivalent to the inverse Fourier transform of the power spectrum $|X(\omega)|^2$ of $x(t)$, where t is replaced with τ . By utilizing the relationship, find the auto-correlation function of the impulse response of the system from the squared amplitude response $|H(\omega)|^2$. (8 points)
- (4) Suppose that $g = 1$ for the system of equation (a) and a complex exponential signal $x(t) = e^{j\omega_0 t} = e^{j2\pi f_0 t}$ is input to the system. When the frequency of the complex exponential signal is $f_0 = \frac{2n - 1}{2T}$ [Hz] where n is a positive integer, find the output $y(t)$ of the system. (8 points)
- (5) When $g = 1$, find the frequency amplitude response $|H(\omega)|$ of the system as a function of $\frac{\omega T}{2}$. In addition, show the frequency phase response $\angle H(\omega)$. (8 points)

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Answer Sheets
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Examinee's number

Use a separate answer sheet for each question.



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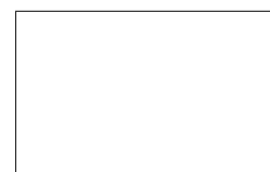


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Answer Sheets
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Answer Sheets
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Calculation Sheets
(2 Sheets in total)

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