Acoustic Engineering / Signal Processing	Example Answers (page 1 of 1 4)	Examinee's number

Use a separate answer sheet for each question.

Question I

Intention of the exam question:
To judge whether examinees understand how to analyze governing equations of single-degree-of-freedom mass-spring-damper systems.

Answer Examples:
(1)
The mechanical impedance is the complex amplitude of driving force divided by that of velocity of a mechanical system. This quantity expresses the difficulty in moving the mechanical system. It has frequency characteristics, i.e., varies with frequency.

(2)

$$R + j\left(\omega M - \frac{K}{\omega}\right)$$

(3)

$$f_S(t) = KA\cos(\omega t + \phi)$$
$$f_D(t) = -RA\omega\sin(\omega t + \phi)$$

(4)

$$F_B = A\sqrt{K^2 + R^2\omega^2}$$

(5)

$$\tau = \sqrt{\frac{\omega_0^4 + \gamma^2\omega^2}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}}$$

Question I (Continued)

(6)
When

$$\omega_0^4 + \gamma^2\omega^2 = \left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2,$$

the relation $\tau = 1$ can be realized. Transforming the equation above, we get

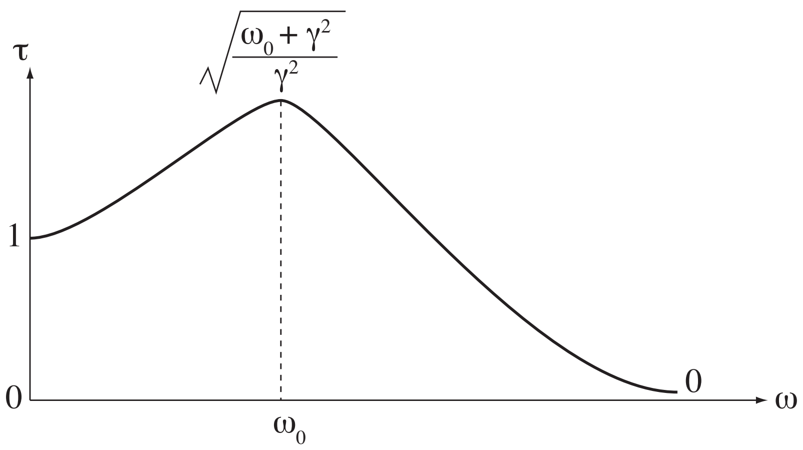
$$\omega_0^2 = \omega^2 - \omega_0^2 \quad \left(\text{when} \quad \omega^2 \geq \omega_0^2 \quad \right),$$
$$\omega^2 = 2\omega_0^2 = 2\frac{K}{M}.$$

From this, adjusting the values of M and K such that

$$\frac{K}{M} \leq \frac{\omega^2}{2},$$

we can realize $\tau \leq 1$.

(7)



Use a separate answer sheet for each question.

Question II (40 points)

(1) Sound pressure level SPL is given by the following.

$$\text{SPL} = 10\log_{10}\left(\frac{0.2/\sqrt{2}}{P_{\text{ref}}^2}\right)^2 = 20log_{10}\left(\frac{2\times10^{-1}}{2\times10^{-5}}\frac{1}{\sqrt{2}}\right) = 20\log_{10}10^4 - 10\log_{10}2 \approx 80-3 = 77 \ \ [\text{dB}]$$

The sound intensity level, IL, is given by the following equation, assuming that the specific acoustic resistance, Z_0, is approximately 400.

$$IL = 10\log_{10}((|p^2/(2Z_0))/I_{\text{ref}}) = 10\log_{10}(\frac{0.04/(2\times400)}{1.0\times10^(-12)} = 10\log_{10}\frac{1}{10^{-8}\times2} \approx 80-3 = 77 \ \ [\text{dB}]$$

(2) Consider a small section on the x-axis with cross-sectional area S, spanning x and $x+dx$. Assume that this small section moves due to the pressure difference between $x=x$ and $x=x+dx$. This driving force is represented by $p(x)S - p(x+dx)S$, but when the second term is expanded using Taylor series, it become $\left[p(x)+\frac{\partial p}{\partial x}dx+\cdots\right]S$. Thus, the driving force is $-\frac{\partial p}{\partial x}dxS$. This force causes the mass $\rho_0 Sdx$ to move with acceleration $\frac{du_x}{dt}$. Here, if we approximate the acceleration as $\frac{du_x}{dt}\approx\frac{\partial u_x}{\partial t}$, we obtain the following equation of motion.

$$\frac{\partial p}{\partial x}\,dxS = -\frac{\partial u_x}{\partial t}\rho_0 dxS, \quad \therefore \frac{\partial p}{\partial x}+\rho_0\frac{\partial u_x}{\partial t} = 0$$

(3) From this equation, the particle velocity can be obtained as follows.

$$\frac{\partial u_x}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}, \quad \therefore u_x = -\frac{1}{\rho_0}\int_{-\infty}^{t}\frac{\partial p}{\partial x}dt \approx -\frac{1}{\rho_0}\int_{-\infty}^{t}\frac{p(x+\Delta x)-p(x)}{\Delta x}dt$$

Therefore, by approximating the partial derivative of sound pressure with respect to x using finite differences and then integrating for time (adding time series data in measurements), we can calculate the particle velocity. Note that to approximate the partial derivative using finite differences, the difference Δx must be sufficiently small compared to the wavelength; otherwise, the error will become large.

(4) The vertical incident sound absorption coefficient α_0 is defined as the ratio of the energy that does not return to the incident side when a plane wave is incident perpendicular to the material. For example, if the amplitude of the incident sound wave is P_i and the amplitude of the reflected sound wave is P_r, then $\alpha_0 = 1-|P_r|^2/|P_i|^2$. A representative measurement method involves introducing a plane wave into an acoustic tube where the diameter (dimension) is sufficiently small compared to the wavelength, and generating standing waves at the front surface of the material. In this case, the amplitude of the antinode of the standing wave is given by P_i+P_r, and the amplitude of the node is P_i-P_r. Therefore, by measuring the ratio of the two, we obtain $n=(P_i-P_r)/(P_i+P_r)$, and thus $|P_r/P_i| = |(1-n)/(1+n)|$ from this, we can obtain $\alpha_0 = 1-|(1-n)/(1+n)|^2$.

Question II (Continued)

(5) The power level PWL of a small sound source in a free field and the sound pressure level L at a distance r from the sound source are related by $L = PWL - 20\ log_{10} r - 11$. The sound pressure level L' of the same sound source with PWL placed at a distance $2\ r$ is $L' = PWL - 20\log_{10}(2\ r) - 11 = L - 20\log_{10} 2$. We can consider the sum of these L and L'. Let the effective sound pressure values be P and P', then the relationship between the levels and the combined level L_{tot} is as follows.

$$P^2 = P_{\mathrm{ref}}^2 \cdot 10^{L/10},\ \ P'^2 = P_{\mathrm{ref}}^2 \cdot 10^{L'/10} = P_{\mathrm{ref}}^2 \cdot 10^{L/10} \cdot 10^{-20\log_{10} 2/10} = P_{\mathrm{ref}}^2 \cdot 10^{L/10}/4$$

$$\therefore P^2 + P'^2 = P_{\mathrm{ref}}^2 \cdot 10^{L/10}(1 + 1/4),\ \ \therefore L_{\mathrm{tot}} = L + 10\log_{10}(5/4)$$

Therefore, compared to L, there will be an increase of $10\log_{10}(5/4)$ (approximately 0.9691) dB.

Acoustic Engineering / Signal Processing	Example Answers (page 5 of 1 4)	Examinee's number

Use a separate answer sheet for each question.

Question III (40 points)

Solve the following discrete-time signal processing problems using only the time domain. Do not solve in the z domain. Note that n represents time, and $u[n]$ is the unit step sequence shown below.

$$u[n] = \begin{cases} 0 & (n < 0) \\ 1 & (n \geq 0) \end{cases}$$

The convolution of sequences $x[n]$ and $y[n]$ is given by the following equation.

$$x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$$

The following formula may also be used. Note that α in the formula is a real number.

$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} \frac{1-\alpha^N}{1-\alpha} & (\alpha \neq 1) \\ N & (\alpha = 1) \end{cases}$	$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \quad (\lvert\alpha\rvert < 1)$
$\sum_{n=k}^{\infty} \alpha^n = \frac{\alpha^k}{1-\alpha} \quad (\lvert\alpha\rvert < 1)$	$\sum_{n=0}^{\infty} n\alpha^n = \frac{\alpha}{(1-\alpha)^2} \quad (\lvert\alpha\rvert < 1)$

(1) Consider a discrete-time linear time-invariant system with impulse response $h_1[n] = \alpha_1^{n-1}u[n-1]$. Show whether this system is causal or not, and explain your reasoning. Note that α_1 is an arbitrary real number. (10 points)

Since the impulse response $h[n] = 0$ for $n < 0$, this system is causal.

Do not write on the back side of the answer sheet, or your answers will not be marked.

Acoustic Engineering / Signal Processing	Example Answers (page 6 of 14)	Examinee's number

Use a separate answer sheet for each question.

Question III (Continued)

(2) Show that the system in the previous question (1) is Bounded-Input Bounded-Output Stability using α_1 and include the explanation process. (5 points)

The condition for Bounded-Input Bounded-Output Stability in the time domain is

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

Therefore, for this system

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} |\alpha_1^{k-1} u[k-1]| = \sum_{k=1}^{\infty} |\alpha_1^{k-1}|$$

Setting $l = k - 1$

$$\sum_{l=0}^{\infty} |\alpha_1|^l = \frac{1}{1 - |\alpha_1|} \quad (|\alpha_1| < 1)$$

Therefore, when $|\alpha_1| < 1$, the system is Bounded-Input Bounded-Output stable; when $|\alpha_1| \geq 1$, the system is Bounded-Input Bounded-Output unstable.

(3) Consider a discrete-time linear time-invariant system with impulse response $h_2[n] = \beta_2^n u[n]$.Calculate the output $y_2[n]$ when the signal $x_2[n] = \alpha_2^n u[n]$ is input to this linear time-invariant system, including the calculation process. Note that α_2 and β_2 are arbitrary real numbers. (10 points)

$$y_2[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} \alpha_2^k u[k] \beta_2^{n-k} u[n-k]$$

$$u[k]u[n-k] = \begin{cases} 1 & (0 \leq k \leq n) \\ 0 & (k < 0, k > n)) \end{cases}$$

From this

$$y_2[n] = \sum_{k=0}^{n} \alpha_2^k \beta_2^{n-k} = \beta_2^n \sum_{k=0}^{n} \left(\frac{\alpha_2}{\beta_2}\right)^k$$

From the formula

$$y_2[n] = \begin{cases} \beta_2^n \frac{1-\left(\frac{\alpha_2}{\beta_2}\right)^{n+1}}{1-\left(\frac{\alpha_2}{\beta_2}\right)} u[n] & (\alpha_2 \neq \beta_2) \\ \beta_2^n (n+1) u[n] & (\alpha_2 = \beta_2) \end{cases}$$

Question III (Continued)

(4) Consider a discrete-time linear time-invariant system whose impulse response is $h_3[n] = \alpha_3^{-n} u[-n]$. Show the output $y_3[n]$ for $n \leq 0$ when the signal $x_3[n] = \alpha_3^n u[n]$ is input to this linear time-invariant system, including the calculation process. Note that α_3 is a real number such that $0 < \alpha_3 < 1$. (Pay attention to $u[-n]$.) (5 points)

$$y_3[n] = \sum_{k=-\infty}^{\infty} x_3[k] h_3[n-k] = \sum_{k=-\infty}^{\infty} \alpha_3^k u[k] \alpha_3^{-(n-k)} u[-(n-k)] = \sum_{k=-\infty}^{\infty} \alpha_3^{-n} \alpha_3^{2k} u[k] u[k-n]$$

When $n \leq 0$

$$u[k]u[k-n] = \begin{cases} 1 & (k \geq 0) \\ 0 & (k < 0)) \end{cases}$$

From the formula

$$y_3[n] = \sum_{k=0}^{\infty} \alpha_3^{-n} \alpha_3^{2k} = \alpha_3^{-n} \sum_{k=0}^{\infty} (\alpha_3^2)^k = \frac{\alpha_3^{-n}}{1 - \alpha_3^2}$$

(5) Given the system and input in the previous question (4), show the output $y_3[n]$ for $n > 0$ including the calculation process. (5 points)

When $n > 0$

$$u[k]u[k-n] = \begin{cases} 1 & (n \leq k) \\ 0 & (n > k) \end{cases}$$

Therefore

$$y_3[n] = \sum_{k=n}^{\infty} \alpha_3^{-n} \alpha_3^{2k} = \alpha_3^{-n} \frac{\alpha_3^{2n}}{1 - \alpha_3^2} = \frac{\alpha_3^n}{1 - \alpha_3^2}$$

(6) Given the system and input in the previous questions (4) and (5), show the output $y_3[n]$ for all n in a single equation. Do not use case distinctions for n. Also, show whether the system is causal or not, and explain your reasoning. (5 points)

$$y_3[n] = \frac{\alpha_3^{|n|}}{1 - \alpha_3^2}$$

Since $h_3[n] = \alpha_3^{-n} u[-n]$, $h_3[n] \neq 0$ for $n < 0$. Therefore, this system is non-causal.

Acoustic Engineering / Signal Processing	Example Answers (page 8 of 14)	Examinee's number

Use a separate answer sheet for each question.

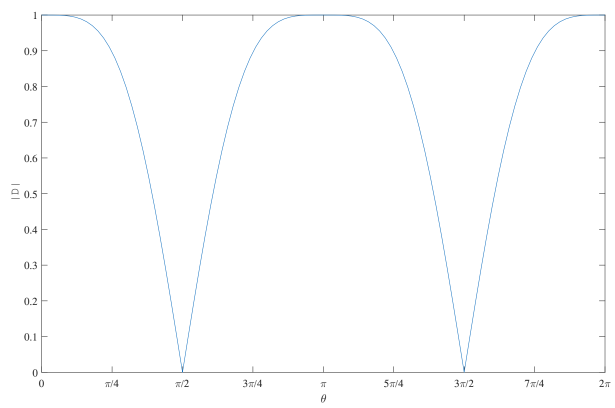
Question IV (40 points)

(1) $\phi_0(t) = \sqrt{2}KPe^{j\omega t}$

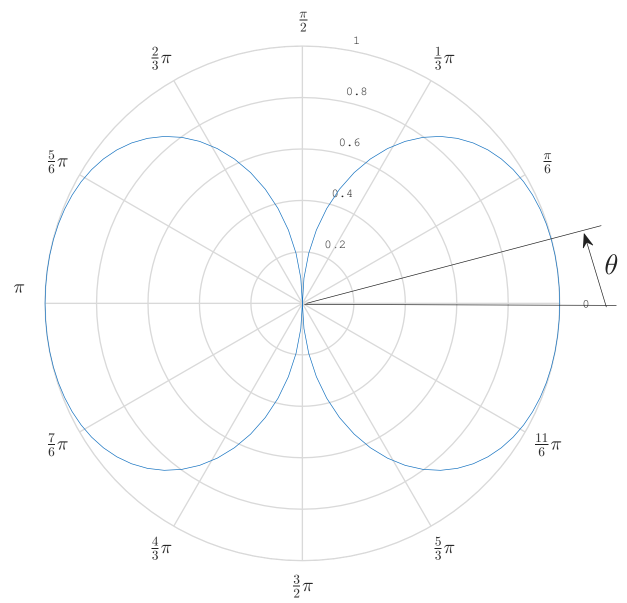
(2) On Microphone element A , $-\frac{d}{2c}\cos\theta$
On Microphone element B ,$\frac{d}{2c}\cos\theta$

(3) $\phi(t) = \sqrt{2}PK\{e^{j\omega(t+\frac{d\cos\theta}{2c})} - e^{j\omega(t-\frac{d\cos\theta}{2c})}\}$
$= \sqrt{2}PK\{e^{j\omega(t+\frac{kd}{2}\cos\theta)} - e^{j\omega(t-\frac{kd}{2}\cos\theta)}\}$

(4) $|\frac{\phi(t)}{\phi_O(t)}|$
$= |e^{\frac{kd}{2}\cos\theta} - e^{-\frac{kd}{2}\cos\theta}|$
$= |2\sin(\frac{kd}{2}\cos\theta)|$
Normalizing the above equation by the maximum value gives us the following D
$D = |\sin(\frac{kd}{2}\cos\theta)|$



Or



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問題 V

(1) **Diffuse Sound Field:** A sound field in which the energy density is uniform at all points, and at any given point, sound energy arrives with equal probability from all directions.
Reverberation Time: The time required for the sound energy in the field to decay to one-millionth of its initial value after the sound source has stopped, corresponding to a decay of 60 dB in level.

(2) The decay of the total acoustic energy density in the sound field is expressed by the ordinary differential equation $V\frac{dE}{dt} = -\frac{Ec}{4}S\overline{\alpha}$. Solving for E gives $E(t) = E_0 e^{-\frac{cS\overline{\alpha}}{4V}t}$, where E_0 is the acoustic energy density at the time $t = 0$ s when the sound source stops. The reverberation time is defined as the time T at which $E(T) = E_0 10^{-6} = E_0 e^{-\frac{cS\overline{\alpha}}{4V}T}$, and solving for T yields Sabine's reverberation formula:

$$T = \frac{24 \ln 10}{c}\frac{V}{S\overline{\alpha}} = \frac{KV}{S\overline{\alpha}}. \quad (1)$$

問題 V(Continued)

(3) When the sound source stops at time $t = 0$ s, let the acoustic energy density in the room be E_0. By time $t = t'$, the sound has propagated ct' meters, during which it is reflected on average $ct'S/4V$ times within the room. Since a fraction $\overline{\alpha}$ of the energy is absorbed with each reflection, the remaining energy in the sound field at time $t = t'$ is $E(t') = E_0(1-\overline{\alpha})^{\frac{ct'S}{4V}} = E_0 e^{\ln(1-\overline{\alpha})\frac{ct'S}{4V}}$. The reverberation time is defined as the time T at which $E(T) = E_0 10^{-6} = E_0 e^{\ln(1-\overline{\alpha})\frac{cS}{4V}T}$, and solving for T yields Eyring's reverberation formula:

$$T = \frac{24\ln 10}{c}\frac{V}{-S\ln(1-\overline{\alpha})} = \frac{KV}{-S\ln(1-\overline{\alpha})}. \tag{2}$$

(4-1) Noting that $2.88 = \frac{288}{100} = \frac{8\times 9}{25}$, the ratio V/S is calculated as

$$\frac{V}{S} = \frac{8\times 9\times \frac{8\times 9}{25}}{2(8\times 9 + 8\times\frac{8\times 9}{25} + 9\times\frac{8\times 9}{25})} \tag{3}$$

$$= \frac{8\times 9}{2(25+8+9)} \tag{4}$$

$$= \frac{6}{7}. \tag{5}$$

Also, since $K = 0.161 = \frac{161}{1000}$,

$$K\frac{V}{S} = \frac{161}{1000}\times\frac{6}{7} = \frac{69}{500}. \tag{6}$$

(a) From Sabine's reverberation formula,

$$\overline{\alpha} = \frac{KV}{ST} = \frac{69}{500\times 0.4} = \frac{69}{200} = 0.345 =: \overline{\alpha}_{\mathrm{S}}. \tag{7}$$

(4-2) From Eyring's reverberation formula and the given approximation,

$$\overline{\alpha} + \frac{\overline{\alpha}^2}{2} = \frac{KV}{ST} = \frac{69}{500\times 0.4} = \frac{69}{200}. \tag{8}$$

Rearranging gives

$$\overline{\alpha}^2 + 2\overline{\alpha} - \frac{69}{100} = 0. \tag{9}$$

Through completing the square,

$$(\overline{\alpha}+1)^2 - 1 - \frac{69}{100} = (\overline{\alpha}+1)^2 - \frac{169}{100} = (\overline{\alpha}+1)^2 - \left(\frac{13}{10}\right)^2 \tag{10}$$

$$= \left(\overline{\alpha}+1+\frac{13}{10}\right)\left(\overline{\alpha}+1-\frac{13}{10}\right) \tag{11}$$

$$= \left(\overline{\alpha}+\frac{23}{10}\right)\left(\overline{\alpha}-\frac{3}{10}\right) = 0. \tag{12}$$

Since $\overline{\alpha} \geqq 0$, we obtain $\overline{\alpha} = 0.3 =: \overline{\alpha}_{\mathrm{E}}$.

(4-3) The mean absorption coefficient $\overline{\alpha}_{\mathrm{S}}$ calculated using Sabine's formula is an overestimation compared to $\overline{\alpha}_{\mathrm{E}}$ from Eyring's formula. Therefore, if we adopt $\overline{\alpha}_{\mathrm{S}}$, the actual reverberation time will be theoretically shorter than 0.4 s.

問題 V(Continued)

(4) First, the required equivalent absorption area is

$$A = \frac{KV}{T} = \frac{161}{1000} \times \frac{100}{46} \times 8 \times 9 \times \frac{8 \times 9}{25} = \frac{7 \times 16 \times 81}{5}\frac{1}{25} = \frac{9072}{125}. \tag{13}$$

It should be checked whether this equivalent absorption area can be achieved through ceiling absorption alone.

(5-1) The equivalent absorption area according to Sabine's formula is

$$A = S_{\text{ceiling}}\alpha_{\text{ceiling}} = 8 \times 9 \times \frac{84}{100} = \frac{8 \times 9 \times 21}{25} = \frac{7560}{125}. \tag{14}$$

Since even a full coverage of this area is insufficient, a design based on ceiling-only absorption is not feasible.

(5-2) If the entire ceiling is covered with absorptive material, the mean absorption coefficient becomes

$$\overline{\alpha} = \frac{8 \times 9}{2(8 \times 9 + (8+9)\frac{8\times 9}{25})} \times \frac{84}{100} \tag{15}$$

$$= \frac{25}{2(25 + 8 + 9)} \times \frac{84}{100} = \frac{1}{4}. \tag{16}$$

The equivalent absorption area according to Eyring's formula is

$$A = -S\ln(1 - \overline{\alpha}) = S\left(\overline{\alpha} + \frac{\overline{\alpha}^2}{2}\right) \tag{17}$$

$$= 2(8 \times 9 + (8+9)\frac{8 \times 9}{25}) \times \frac{9}{32} \tag{18}$$

$$= 2 \times 8 \times 9 \times \frac{42}{25} \times \frac{9}{32} \tag{19}$$

$$= \frac{21 \times 81}{25} = \frac{8505}{125}. \tag{20}$$

Here again, a ceiling-only absorption design is not feasible.

Acoustic Engineering / Signal Processing

Example Answers

Examinee's number

Use a separate answer sheet for each question.

Question VI

(1)

$$h(n) = \frac{1}{5}\sum_{i=0}^{4}\delta[n-i]$$

(2)

$$y[n] = \frac{1}{5}\sum_{i=0}^{4}x[n-i]$$

$$y[n-1] = \frac{1}{5}\sum_{i=0}^{4}x[n-i-1]$$

(3)

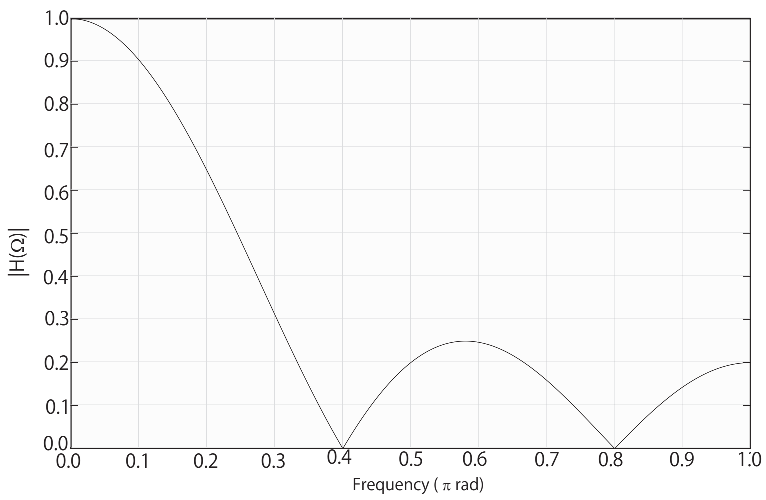
$$H(\Omega) = \frac{1}{5}\left\{1+e^{-j\Omega}+e^{-j2\Omega}+e^{-j3\Omega}+e^{-j4\Omega}\right\}$$

(4)

$$|H(\Omega)| = \frac{1}{5}|1+2\cos 2\Omega+2\cos\Omega|$$

$$\angle H(\Omega) = \begin{cases} -2\Omega \quad , & \text{for } \Omega \le \frac{2}{5}\pi \\ \pi-2\Omega, & \text{for } \frac{2}{5}\pi \le \Omega \le \frac{4}{5}\pi \\ 2\pi-2\Omega, & \text{for } \Omega \ge \frac{4}{5}\pi \end{cases}$$

(5)



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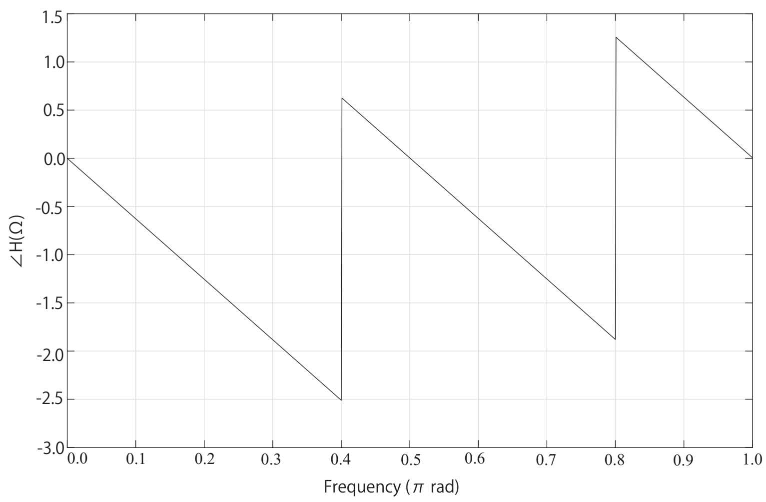
Question VI(Continued)

(6)



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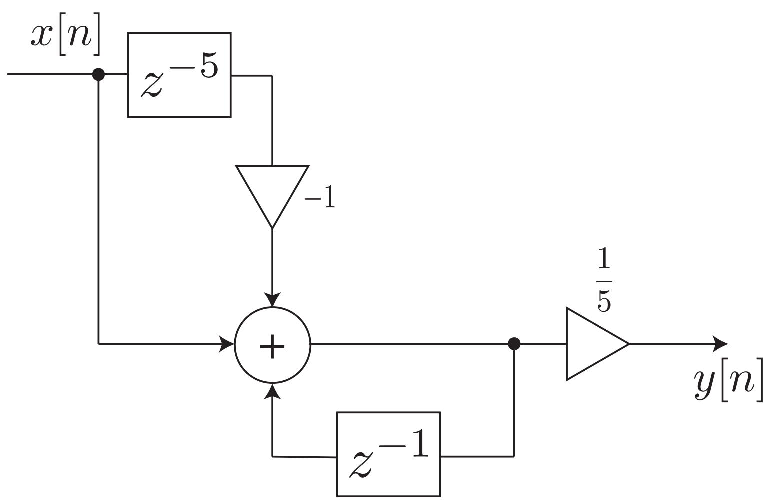

Acoustic Engineering / Signal Processing	Example Answers (page 1 4 of 1 4)	Examinee's number

Use a separate answer sheet for each question.

Question VII

(1) $X'(\omega) = e^{-j\omega s}X(\omega)$

(2) $H(\omega) = 1 + ge^{-j\omega T}$

(3) $\phi(t) = (1+g^2)\delta(t) + g\{\delta(t-T) + \delta(t+T)\}$

(4) 0

(5)

$$
\begin{aligned}
|H(\omega)| &= 2|\cos\left(\frac{\omega T}{2}\right)| \\
\angle H(\omega) &= -\frac{\omega T}{2} \qquad (0 \le \omega < \frac{\pi}{T}) \\
&= -\frac{\omega T}{2} + \pi \qquad (\frac{\pi}{T} < \omega < \frac{3\pi}{T}) \\
&= -\frac{\omega T}{2} + 2\pi \qquad (\frac{3\pi}{T} < \omega \le \frac{4\pi}{T})
\end{aligned}
$$

Acoustic Engineering / Signal Processing

Intention of the Exam Questions

Question I

To judge whether examinees understand how to analyze governing equations of single-degree-of-freedom mass-spring-damper systems.

Question II

This problem is designed to assess whether examinees understand the terminology related to sound waves propagating through a medium, and whether they can represent these characteristics mathematically and handle them numerically.

Question III

This question aims to assess the level of understanding of signal processing required for the master's degree research.

Question IV

To assess whether the examinees understand how a simple microphone array works.

Question V

This problem assesses the ability to concisely explain the derivation process of fundamental formulas in room acoustics and to correctly apply the formulas to evaluate results.

Question VI

This question aims to assess the level of understanding of digital signal processing required for the master's degree research.

Question VII

This question aims to assess the level of understanding of acoustic signal processing required for the master's degree research.